HULL'S
MENTAL ARITHMETIC

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Hull's Mental Arithmetic.

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MENTAL

ARITHMETIC.

BASED ON

ANALYSIS AND INDUCTION.

BY

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PREFACE.

THE object of this book is to give the pupil that *power of analysis* which is neither furnished by a course in Written Arithmetic nor acquired by the study of Algebra, and yet is essential to the easy mastery of mathematics.

A mistake is sometimes made in regarding Mental Arithmetic as Oral Arithmetic merely. This places the process, or form of solution, above the principle that underlies a system of mental arithmetic, and leads to false methods of instruction. Though mental arithmetic is generally oral, and should be eminently practical, yet its essential feature is not necessarily either. The end to be attained is *analytic power.* This is accomplished by analysis, comparison, and induction.

The author has tried to make the work as practical as possible, while keeping steadily in view the distinctive feature of the subject; also to make the course comprehensive, logical, and progressive.

Great care has been taken in the construction of problems, to make them neither too difficult, thus requiring the aid of written characters, nor too easy, thus depriving the pupil of the power that comes from patient effort.

<div align="right">GEORGE W. HULL.</div>

MILLERSVILLE, PA., }
  Sept. 1, 1896.   }

INTRODUCTION.

ATTENTION is called to the following suggestions.

Elementary Results.—It is readily perceived that there is a number of elementary results which we have no means of deriving except through the process of counting. Thus, if we are asked to add 4 units to 5 units, we have no means of obtaining the result except to count from 5 to 9.

These elementary results are the *sums of the addition table and the products of the multiplication table, with their corresponding differences and quotients.* No great progress can be made in arithmetic until these elementary results are mastered.

It should be remembered that each elementary result is formed of *only* two numbers of one figure each. Thus, $4 + 5 = 9$ and $4 \times 5 = 20$ are elementary results; but $15 + 3 = 18$, $4 + 8 + 3 = 15$, and $3 \times 5 \times 6 = 90$, are *not* elementary results.

It should also be remembered that the highest possible elementary sum is $9 + 9 = 18$, and the highest possible elementary product is $9 \times 9 = 81$. This means, not that the addition table cannot extend above $9 + 9 = 18$, nor the multiplication table above $9 \times 9 = 81$, but that there is no absolute necessity for them beyond these limits.

The first step, therefore, in Arithmetic is to familiarize the pupil with these elementary results. Since a child, in acquiring knowledge, deals first with the object, then with a mental pic-

ture of the object, and finally with the abstract idea, this order should be followed in teaching these elementary results: *first,* with objects; *second,* without objects; *third,* with figures.

METHODS OF RECITATION.

There are two methods now in general use of conducting a recitation in mental arithmetic. They are as follows:

1. **Without the Book.**—By this method, the teacher reads the problems and assigns them promiscuously. The pupil to whom a problem is assigned rises, repeats the problem, and solves it without the aid either of the book or of written characters. If any mistakes are made in the solution, they are corrected either by the class or by the teacher.

2. **With the Book.**—By this method, the pupil is permitted to have the book. The problems are assigned promiscuously as before. The pupil to whom a problem is assigned rises with the book in hand, and solves the problem without the aid of written characters. Corrections are made as in the other method.

In both these methods the entire class may solve the problems silently, and, when all the members of the class, or nearly all, have acquired the result, the teacher may designate some one to give the result and solve the problem.

MENTAL ARITHMETIC.

I. ELEMENTARY RESULTS.

I. ELEMENTARY SUMS.

1. RUTH had 3 cents, and Mary gave her 2 more; how many cents had she then?

SOLUTION.—If Ruth had 3 cents, and Mary gave her 2 more, she then had 3 cents and 2 cents, which are 5 cents.

2. John has 2 cents, and Louis has 1 cent; how many cents do they together have?

3. Harry had 4 apples, and bought 2 more; how many apples did he then have?

4. There are 5 boys in one class, and 2 in another; how many boys in both classes?

5. Harold had 3 marbles, and bought 3 more; how many marbles had he then?

6. A farmer had 4 horses, and bought 3 more; how many horses had he then?

7. There are 5 roses on one bush, and 3 on another; how many roses are there on both bushes?

8. A slate cost 7 cents, and a pencil 2 cents; how many cents did both cost?

9. Gilbert had 4 cherries, and his brother gave him 4 more; how many cherries did he then have?

10. A boy rode 4 miles, and walked 5; how many miles did he travel?

11. How many are 3 and 6?

FORM OF RECITATION.—3 and 6 are 9.

12. How many are—

2 and 2?	6 and 2?	2 and 3?	6 and 3?
3 and 2?	7 and 2?	3 and 3?	7 and 3?
4 and 2?	8 and 2?	4 and 3?	8 and 3?
5 and 2?	9 and 2?	5 and 3?	9 and 3?

13. James had 5 cents, and earned 3 more; how many cents did he then have?

14. Maud had 5 pinks, and gathered 4 more; how many pinks did she then have?

15. Esther paid 7 cents for a card, and 5 cents for a pencil; how many cents did both cost?

16. How many are—

2 and 4?	6 and 4?	2 and 5?	6 and 5?
3 and 4?	7 and 4?	3 and 5?	7 and 5?
4 and 4?	8 and 4?	4 and 5?	8 and 5?
5 and 4?	9 and 4?	5 and 5?	9 and 5?

17. A pound of sugar cost 6 cents, and a quart of vinegar 5 cents; how many cents did both cost?

18. Henry has 6 dollars in gold, and 7 dollars in silver; how many dollars has he of both kinds?

19. How many are—

2 and 6?	6 and 6?	2 and 7?	6 and 7?
3 and 6?	7 and 6?	3 and 7?	7 and 7?
4 and 6?	8 and 6?	4 and 7?	8 and 7?
5 and 6?	9 and 6?	5 and 7?	9 and 7?

20. James had 8 cents, and found 6 more; how many cents did he then have?

21. Ross caught 7 fish, and Horace caught 8; how many fish did they both catch?

22. How many are—

2 and 8?	6 and 8?	2 and 9?	6 and 9?
3 and 8?	7 and 8?	3 and 9?	7 and 9?
4 and 8?	8 and 8?	4 and 9?	8 and 9?
5 and 8?	9 and 8?	5 and 9?	9 and 9?

23. Helen gave 5 cents for an orange, and 9 cents for a slate; how much did she pay for both?

24. A farmer sold 6 bushels of apples to one man, and 9 bushels to another; how many bushels did he sell to both?

The process of finding the sum of two or more numbers is called **Addition.**

The symbol + is read *plus.* It indicates that the numbers between which it is placed are to be added.

The symbol = is read *equal* or *equals.* Thus, 5 + 3 = 8 is read 5 *plus* 3 *equal* 8.

25. What additions of two figures will produce 4? 5? 6?

26. What additions of two figures will produce 7? 8? 9?

27. What additions of two figures will produce 10? 11? 12?

28. What additions of two figures will produce 13? 14?

29. What additions of two figures will produce 15? 16?

30. What addition of two figures will produce 17? 18?

The simplest elementary sum is 1 + 1. The following are the eight most difficult elementary sums:

31. How many are—

| 8 and 9? | 6 and 9? | 8 and 5? | 7 and 6? |
| 7 and 8? | 7 and 9? | 8 and 6? | 9 and 5? |

TABLE OF ELEMENTARY SUMS.

1 + 2 = 3	1 + 3 = 4	1 + 4 = 5	1 + 5 = 6
2 + 2 = 4	2 + 3 = 5	2 + 4 = 6	2 + 5 = 7
3 + 2 = 5	3 + 3 = 6	3 + 4 = 7	3 + 5 = 8
4 + 2 = 6	4 + 3 = 7	4 + 4 = 8	4 + 5 = 9
5 + 2 = 7	5 + 3 = 8	5 + 4 = 9	5 + 5 = 10
6 + 2 = 8	6 + 3 = 9	6 + 4 = 10	6 + 5 = 11
7 + 2 = 9	7 + 3 = 10	7 + 4 = 11	7 + 5 = 12
8 + 2 = 10	8 + 3 = 11	8 + 4 = 12	8 + 5 = 13
9 + 2 = 11	9 + 3 = 12	9 + 4 = 13	9 + 5 = 14
1 + 6 = 7	1 + 7 = 8	1 + 8 = 9	1 + 9 = 10
2 + 6 = 8	2 + 7 = 9	2 + 8 = 10	2 + 9 = 11
3 + 6 = 9	3 + 7 = 10	3 + 8 = 11	3 + 9 = 12
4 + 6 = 10	4 + 7 = 11	4 + 8 = 12	4 + 9 = 13
5 + 6 = 11	5 + 7 = 12	5 + 8 = 13	5 + 9 = 14
6 + 6 = 12	6 + 7 = 13	6 + 8 = 14	6 + 9 = 15
7 + 6 = 13	7 + 7 = 14	7 + 8 = 15	7 + 9 = 16
8 + 6 = 14	8 + 7 = 15	8 + 8 = 16	8 + 9 = 17
9 + 6 = 15	9 + 7 = 16	9 + 8 = 17	9 + 9 = 18

II. ELEMENTARY DIFFERENCES.

1. Mary had 5 cents, and lost 2 of them ; how many cents had she remaining?

SOLUTION.—If Mary had 5 cents, and lost 2 of them, she had remaining the difference between 5 cents and 2 cents, which is 3 cents. Therefore, etc.

2. Alice had 4 roses, and gave 2 of them away; how many roses had she remaining?

3. There were 5 birds on a tree, and 3 of them flew away; how many birds remained?

4. In a class of 9 pupils, 4 of them were absent; how many pupils were present?

5. Emily had 11 pinks, and gave her sister 6 of them; how many pinks had she remaining?

6. How many are 3 from 6?

FORM OF RECITATION.—3 from 6 are 3.

7. How many are—

2 from 2?	2 from 7?	3 from 3?	3 from 8?
2 from 3?	2 from 8?	3 from 4?	3 from 9?
2 from 4?	2 from 9?	3 from 5?	3 from 10?
2 from 5?	2 from 10?	3 from 6?	3 from 11?
2 from 6?	2 from 11?	3 from 7?	3 from 12?

8. John had 10 apples, and gave 4 of them away; how many apples had he remaining?

9. Harold paid 12 cents for a book, and 4 cents for a pencil; how much more did he pay for the book than for the pencil?

10. A boy had 13 marbles, and lost 5 of them; how many marbles had he remaining?

11. Amos planted 14 trees, and 5 of them died; how many of the trees grew?

12. How many are—

4 from 4?	4 from 9?	5 from 5?	5 from 10?
4 from 5?	4 from 10?	5 from 6?	5 from 11?
4 from 6?	4 from 11?	5 from 7?	5 from 12?
4 from 7?	4 from 12?	5 from 8?	5 from 13?
4 from 8?	4 from 13?	5 from 9?	5 from 14?

13. There were 12 birds on a tree, and 6 of them flew away; how many birds remained?

14. Harry had 13 marbles, and lost 6 of them; how many marbles had he remaining?

15. James had 14 apples, and gave 7 of them away; how many apples had he remaining?

16. Arthur picked 15 quarts of berries, and sold 7 of them; how many quarts had he remaining?

17. William had 14 cents, and spent 6 of them; how many cents had he remaining?

18. How many are—

6 from 6?	6 from 11?	7 from 7?	7 from 12?
6 from 7?	6 from 12?	7 from 8?	7 from 13?
6 from 8?	6 from 13?	7 from 9?	7 from 14?
6 from 9?	6 from 14?	7 from 10?	7 from 15?
6 from 10?	6 from 15?	7 from 11?	7 from 16?

19. A farmer had 12 sheep, and sold 8 of them; how many sheep had he remaining?

20. Anna had 15 dollars, and spent 9 of them; how many dollars had she remaining?

21. Frank had 16 marbles, and lost 8 of them; how many marbles had he remaining?

22. A drover had 17 cattle, and sold 9 of them; how many cattle had he remaining?

23. How many are—

8 from 8?	8 from 13?	9 from 9?	9 from 14?
8 from 9?	8 from 14?	9 from 10?	9 from 15?
8 from 10?	8 from 15?	9 from 11?	9 from 16?
8 from 11?	8 from 16?	9 from 12?	9 from 17?
8 from 12?	8 from 17?	9 from 13?	9 from 18?

24. A boy had 15 marbles, and lost 8 of them; how many marbles had he remaining?

25. A lady had 13 dollars, and spent 9 of them; how many dollars had she remaining?

26. Charles had 14 oranges, and gave 8 of them to his brother; how many oranges had he remaining?

27. A drover had 16 horses, and sold 9 of them; how many horses had he remaining?

28. James had 18 quarts of berries, and sold 9 of them; how many quarts had he remaining?

29. How many must be added to 8 to make 11? To make 15? To make 13? To make 17?

30. How many must be added to 9 to make 13? To make 11? To make 18? To make 16? To make 17?

31. How many are 12 less 7? 14 less 7? 16 less 7? 15 less 7? 13 less 7? 10 less 7?

32. How many are 18 less 9? 17 less 9? 16 less 9? 15 less 9? 14 less 9? 13 less 9?

33. How many are 5 from 12? 6 from 13? 7 from 14?

The process of finding the difference between two numbers is called Subtraction.

The symbol − is read *minus.* It indicates that the number after it is to be taken from the number before it. Thus, 8 − 5 = 3 is read 8 *minus* 5 *equal* 3.

TABLE OF ELEMENTARY DIFFERENCES.

2 − 2 = 0	3 − 3 = 0	4 − 4 = 0	5 − 5 = 0
3 − 2 = 1	4 − 3 = 1	5 − 4 = 1	6 − 5 = 1
4 − 2 = 2	5 − 3 = 2	6 − 4 = 2	7 − 5 = 2
5 − 2 = 3	6 − 3 = 3	7 − 4 = 3	8 − 5 = 3
6 − 2 = 4	7 − 3 = 4	8 − 4 = 4	9 − 5 = 4
7 − 2 = 5	8 − 3 = 5	9 − 4 = 5	10 − 5 = 5
8 − 2 = 6	9 − 3 = 6	10 − 4 = 6	11 − 5 = 6
9 − 2 = 7	10 − 3 = 7	11 − 4 = 7	12 − 5 = 7
10 − 2 = 8	11 − 3 = 8	12 − 4 = 8	13 − 5 = 8
11 − 2 = 9	12 − 3 = 9	13 − 4 = 9	14 − 5 = 9
6 − 6 = 0	7 − 7 = 0	8 − 8 = 0	9 − 9 = 0
7 − 6 = 1	8 − 7 = 1	9 − 8 = 1	10 − 9 = 1
8 − 6 = 2	9 − 7 = 2	10 − 8 = 2	11 − 9 = 2
9 − 6 = 3	10 − 7 = 3	11 − 8 = 3	12 − 9 = 3
10 − 6 = 4	11 − 7 = 4	12 − 8 = 4	13 − 9 = 4
11 − 6 = 5	12 − 7 = 5	13 − 8 = 5	14 − 9 = 5
12 − 6 = 6	13 − 7 = 6	14 − 8 = 6	15 − 9 = 6
13 − 6 = 7	14 − 7 = 7	15 − 8 = 7	16 − 9 = 7
14 − 6 = 8	15 − 7 = 8	16 − 8 = 8	17 − 9 = 8
15 − 6 = 9	16 − 7 = 9	17 − 8 = 9	18 − 9 = 9

III. ELEMENTARY PRODUCTS.

1. If 1 orange costs 3 cents, what will 2 oranges cost?

SOLUTION.—If 1 orange costs 3 cents, 2 oranges will cost 2 times 3 cents, which are 6 cents. Therefore, etc.

2. What will 2 apples cost, at 2 cents apiece?

3. What will 2 melons cost, at 5 cents apiece?

4. What will 2 hats cost, at 4 dollars apiece?

5. What will 2 sheep cost, at 7 dollars apiece?

6. How many are—

2 times 1?	2 times 4?	2 times 7?
2 times 2?	2 times 5?	2 times 8?
2 times 3?	2 times 6?	2 times 9?

7. What will 3 balls cost, at 5 cents apiece?

8. What will 3 coats cost, at 7 dollars apiece?

9. What will 3 sheep cost, at 6 dollars apiece?

10. What will 3 pencils cost, at 4 cents apiece?

11. How many are—

3 times 1?	3 times 4?	3 times 7?
3 times 2?	3 times 5?	3 times 8?
3 times 3?	3 times 6?	3 times 9?

12. What will 4 books cost, at 5 dollars apiece?

13. What will 4 tops cost, at 8 cents apiece?

14. What will 4 fur caps cost, at 6 dollars apiece?

15. What will 4 guns cost, at 9 dollars apiece?

16. How many are—

4 times 1?	4 times 4?	4 times 7?
4 times 2?	4 times 5?	4 times 8?
4 times 3?	4 times 6?	4 times 9?

17. What will 5 cards cost, at 6 cents apiece?

18. What will 5 melons cost, at 7 cents apiece?

19. What will 5 tables cost, at 8 dollars apiece?

20. What will 5 pigeons cost, at 9 cents apiece?

21. How many are—

5 times 1?	5 times 4?	5 times 7?
5 times 2?	5 times 5?	5 times 8?
5 times 3?	5 times 6?	5 times 9?

22. What will 6 cakes cost, at 4 cents apiece?

23. What will 6 loaves cost, at 6 cents apiece?

24. What will 6 melons cost, at 7 cents apiece?

25. What will 6 balls cost, at 8 cents apiece?

26. How many are—

6 times 1?	6 times 4?	6 times 7?
6 times 2?	6 times 5?	6 times 8?
6 times 3?	6 times 6?	6 times 9?

27. What will 7 pictures cost, at 8 cents apiece?

28. What will 7 books cost, at 6 cents apiece?

29. What will 7 vases cost, at 7 dollars apiece?

30. What will 7 hats cost, at 5 dollars apiece?

31. How many are—

7 times 1?	7 times 4?	7 times 7?
7 times 2?	7 times 5?	7 times 8?
7 times 3?	7 times 6?	7 times 9?

32. What will 8 dogs cost, at 5 dollars apiece?

33. What will 8 sheep cost, at 6 dollars apiece?

34. What will 8 pigs cost, at 7 dollars apiece?

35. What will 8 calves cost, at 8 dollars apiece?

36. How many are—

8 times 1?	8 times 4?	8 times 7?
8 times 2?	8 times 5?	8 times 8?
8 times 3?	8 times 6?	8 times 9?

37. What will 9 oranges cost, at 5 cents apiece?

38. What will 9 citrons cost, at 6 cents apiece?

39. What will 9 melons cost, at 8 cents apiece?

40. What will 9 squashes cost, at 9 cents apiece?

41. How many are—

9 times 1?	9 times 4?	9 times 7?
9 times 2?	9 times 5?	9 times 8?
9 times 3?	9 times 6?	9 times 9?

42. A man walks 5 miles an hour; how far can he walk in 6 hours?

43. At 7 cents a quart, what will 6 quarts of berries cost?

44. A girl had 6 roses, and 7 times as many pinks; how many pinks had she?

45. How far will a man travel in 8 hours, at the rate of 6 miles an hour?

46. James earns 3 dollars a day; how much will he earn in 8 days?

47. What will 9 yards of ribbon cost, at 8 cents a yard?

48. What will 9 yards of cloth cost, at 9 dollars a yard?

49. In an orchard there are 7 rows of trees, with 8 trees in each row; how many trees in the orchard?

50. If 6 men can do a piece of work in 7 days, how long will it take 1 man to do it?

51. If 8 men can mow a field in 9 days, how long will it take 1 man to mow it?

52. How much will 7 pounds of sugar cost, at 7 cents a.pound? At 8 cents a pound?

53. What will 9 barrels of apples cost, at 5 dollars a barrel? At 6 dollars a barrel?

The process of taking one number as many times as there are units in another is called **Multiplication.**

The symbol × is read *times* or *multiplied by.* Thus, $8 \times 5 = 40$ is read 8 *times* 5 *equal* 40, or 8 *multiplied by* 5 *equals* 40.

TABLE OF ELEMENTARY PRODUCTS.

$2 \times 1 = 2$	$3 \times 1 = 3$	$4 \times 1 = 4$	$5 \times 1 = 5$
$2 \times 2 = 4$	$3 \times 2 = 6$	$4 \times 2 = 8$	$5 \times 2 = 10$
$2 \times 3 = 6$	$3 \times 3 = 9$	$4 \times 3 = 12$	$5 \times 3 = 15$
$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$
$2 \times 5 = 10$	$3 \times 5 = 15$	$4 \times 5 = 20$	$5 \times 5 = 25$
$2 \times 6 = 12$	$3 \times 6 = 18$	$4 \times 6 = 24$	$5 \times 6 = 30$
$2 \times 7 = 14$	$3 \times 7 = 21$	$4 \times 7 = 28$	$5 \times 7 = 35$
$2 \times 8 = 16$	$3 \times 8 = 24$	$4 \times 8 = 32$	$5 \times 8 = 40$
$2 \times 9 = 18$	$3 \times 9 = 27$	$4 \times 9 = 36$	$5 \times 9 = 45$
$6 \times 1 = 6$	$7 \times 1 = 7$	$8 \times 1 = 8$	$9 \times 1 = 9$
$6 \times 2 = 12$	$7 \times 2 = 14$	$8 \times 2 = 16$	$9 \times 2 = 18$
$6 \times 3 = 18$	$7 \times 3 = 21$	$8 \times 3 = 24$	$9 \times 3 = 27$
$6 \times 4 = 24$	$7 \times 4 = 28$	$8 \times 4 = 32$	$9 \times 4 = 36$
$6 \times 5 = 30$	$7 \times 5 = 35$	$8 \times 5 = 40$	$9 \times 5 = 45$
$6 \times 6 = 36$	$7 \times 6 = 42$	$8 \times 6 = 48$	$9 \times 6 = 54$
$6 \times 7 = 42$	$7 \times 7 = 49$	$8 \times 7 = 56$	$9 \times 7 = 63$
$6 \times 8 = 48$	$7 \times 8 = 56$	$8 \times 8 = 64$	$9 \times 8 = 72$
$6 \times 9 = 54$	$7 \times 9 = 63$	$8 \times 9 = 72$	$9 \times 9 = 81$

IV. Elementary Quotients.

1. At 3 cents apiece, how many oranges can you buy for 15 cents?

SOLUTION.—If 1 orange costs 3 cents, for 15 cents you can buy as many oranges as 3 is contained times in 15, which are 5. Therefore, etc.

2. At 2 cents apiece, how many apples can you buy for 6 cents? For 10 cents?

3. At 2 dollars apiece, how many hats can you buy for 12 dollars? For 16 dollars?

4. At 2 dimes apiece, how many caps can you buy for 14 dimes? For 18 dimes?

5. How many times is 2 contained in 12?

SOLUTION.—2 is contained 6 times in 12, since 6 times 2 are 12.

6. How many times is 2 contained—

| In 4? | In 8? | In 12? | In 16? |
| In 6? | In 10? | In 14? | In 18? |

7. At 3 cents a yard, how many yards of ribbon can you buy for 9 cents? For 12 cents?

8. If a man travels 3 miles an hour, how long will it take him to travel 24 miles? 21 miles?

9. James earns 3 dollars a day; how long will it take him to earn 18 dollars? 27 dollars?

10. How many times is 3 contained—

| In 6? | In 12? | In 18? | In 24? |
| In 9? | In 15? | In 21? | In 27? |

11. At 4 cents apiece, how many oranges can be bought for 12 cents? For 8 cents? For 28 cents?

12. In how many days can a man earn 20 dollars, if he earns 4 dollars a day?

13. At 4 dollars a barrel, how many barrels of apples can be bought for 24 dollars? For 36 dollars? For 32 dollars?

14. How many times is 4 contained—

| In 8? | In 16? | In 24? | In 32? |
| In 12? | In 20? | In 28? | In 36? |

15. At 5 cents apiece, how many pencils can be bought for 15 cents? For 10 cents? For 20 cents?

16. At 5 dollars a head, how many calves can be bought for 25 dollars? For 30 dollars? For 35 dollars?

17. At 5 cents apiece, how many melons can be bought for 40 cents? For 55 cents?

18. How many times is 5 contained—

| In 10? | In 15? | In 20? | In 25? |
| In 30? | In 35? | In 40? | In 45? |

19. At 6 dollars apiece, how many sheep can you buy for 12 dollars? For 18 dollars?

20. At 6 cents a bushel, how many bushels of salt can be bought for 24 cents? For 42 cents?

21. If a pair of shoes costs 6 dollars, how many pairs can be bought for 48 dollars? For 54 dollars? For 36 dollars?

22. How many times is 6 contained—

| In 12? | In 18? | In 24? | In 30? |
| In 36? | In 42? | In 48? | In 54? |

23. How many yards of ribbon, at 7 cents a yard, can be bought for 14 cents? For 21 cents?

24. How many sheep, at 7 dollars a head, can be bought for 28 dollars? For 35 dollars? For 42 dollars?

25. How many melons, at 7 cents apiece, can be bought for 49 cents? For 56 cents? For 63 cents?

26. How many times is 7 contained—

| In 14? | In 21? | In 28? | In 35? |
| In 42? | In 49? | In 56? | In 63? |

27. How many yards of muslin, at 8 cents a yard, can be bought for 24 cents? For 16 cents?

28. At 8 cents a quart, how many quarts of berries can be bought for 32 cents? For 40 cents? For 48 cents?

29. How many pencils, at 8 cents apiece, can be bought for 56 cents? For 64 cents? For 72 cents?

30. How many times is 8 contained—

| In 24? | In 32? | In 40? | In 16? |
| In 72? | In 48? | In 56? | In 64? |

31. At 9 cents each, how many books can be bought for 36 cents? For 18 cents?

32. At 9 dollars each, how many clocks can be bought for 45 dollars? For 27 dollars? For 54 dollars?

33. How many lamps, at 9 dollars each, can be bought for 72 dollars? For 63 dollars? For 81 dollars?

34. How many times is 9 contained—

In 18?	In 63?	In 36?	In 81?
In 45?	In 27?	In 54?	In 72?

35. 8 is how many times 2?

SOLUTION.—8 is 4 times 2, since 4 times 2 are 8.

36. How many times 3 is 6?   12?   18?   21?   27?

37. How many times 5 is 10?   25?   45?   35?   40?

38. How many times 6 is 18?   36?   48?   54?   42?

39. How many times 7 is 21?   49?   63?   56?   42?

40. How many 8's in 16?   32?   40?   72?   64?

41. How many 9's in 18?   27?   36?   72?   81?

The process of finding how many times one number is contained in another is called **Division**.

The symbol ÷ is read *divided by*. Thus, 8 ÷ 4 = 2 is read 8 *divided by* 4 *equals* 2.

TABLE OF ELEMENTARY QUOTIENTS.

2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1	5 ÷ 5 = 1
4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2	10 ÷ 5 = 2
6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3	15 ÷ 5 = 3
8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4	20 ÷ 5 = 4
10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5	25 ÷ 5 = 5
12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6	30 ÷ 5 = 6
14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7	35 ÷ 5 = 7
16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8	40 ÷ 5 = 8
18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9	45 ÷ 5 = 9
6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1	9 ÷ 9 = 1
12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2	18 ÷ 9 = 2
18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3	27 ÷ 9 = 3
24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4	36 ÷ 9 = 4
30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5	45 ÷ 9 = 5
36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6	54 ÷ 9 = 6
42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7	63 ÷ 9 = 7
48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8	72 ÷ 9 = 8
54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9	81 ÷ 9 = 9

II. FUNDAMENTAL PROCESSES.

I. ADDITION.

1. HARRY has 4 cents, Lawrence has 8 cents, and Ralph has 6 cents; how many cents do they together have?

SOLUTION.—Since Harry has 4 cents, Lawrence 8, and Ralph 6, they together have 4 cents, 8 cents, and 6 cents, which are 18 cents.

2. Anna has 6 dollars, Mary has 8, and Ruth has 5; how many dollars do they together have?

3. A man walked 6 miles the first day, 7 miles the second, and 8 miles the third; how far did he walk in the three days?

4. A boy bought three toys,—paying 5 cents for the first, 7 cents for the second, and 9 cents for the third; how much did they all cost?

5. Charles is 8 years old, William is 10, and Henry is 11; what is the sum of their ages?

6. There are 6 horses, 8 cows, and 9 sheep in a field; how many animals are there in the field?

7. Aaron has 6 cents, and Amos has 15; how many cents have they both?

8. George had 16 pigeons, and caught 7 more; how many pigeons had he then?

9. Sarah has 8 books, Amanda has 12, and Margaret has 13; how many books do they together have?

10. A boy found 9 eggs in one nest, 11 in another, and 14 in another; how many eggs did he find?

11. A boy went to a store and bought a pound of sugar for 7 cents, a brush for 15 cents, and a broom for 20 cents; how much did all of them cost?

12. How many are 12 men and 13 men? 14 cents and 16 cents? 18 caps and 12 caps? 16 apples and 18 apples?

13. How many are 17 bushels and 15 bushels? 21 weeks and 16 weeks? 25 yards and 15 yards?

14. How many are 10 birds and 18 birds? 17 acres and 13 acres? 22 marbles and 16 marbles?

15. How many are—

20 + 10?	20 + 40?	30 + 30?	20 + 30?	12 + 10?
20 + 20?	30 + 50?	40 + 30?	70 + 20?	12 + 20?
40 + 50?	60 + 30?	50 + 20?	40 + 40?	12 + 30?

16. How many are—

14 + 4?	16 + 6?	23 + 7?	45 + 9?	23 + 12?
24 + 4?	26 + 6?	33 + 7?	55 + 9?	33 + 12?
34 + 4?	36 + 6?	43 + 7?	65 + 9?	43 + 12?

17. How many are—

24 + 15?	17 + 14?	15 + 17?	19 + 22?	17 + 24?
26 + 17?	18 + 22?	18 + 19?	32 + 14?	16 + 25?
28 + 19?	16 + 45?	16 + 17?	16 + 24?	25 + 25?

SUGGESTION.—A practical method of adding numbers like the above is as follows: Add 28 and 37. 28 and 7 are 35, and 30 are 65.

18. How many are 6 boys, 8 boys, and 10 boys?

19. How many are 7 dollars, 9 dollars, and 4 dollars?

20. How many are 9 hens, 12 hens, and 13 hens?

21. How many are 8 pens, 12 pens, and 14 pens?

22. How many are 13 men, 14 men, and 15 men?

23. How many are 7 trees, 16 trees, and 12 trees?

24. How many are 10 pears, 12 pears, and 18 pears?

25. How many are 22 cents, 23 cents, and 25 cents?

26. How many are—

2 + 2 + 2?	5 + 5 + 5?	8 + 8 + 8?	11 + 11 + 11?
3 + 3 + 3?	6 + 6 + 6?	9 + 9 + 9?	12 + 12 + 12?
4 + 4 + 4?	7 + 7 + 7?	10 + 10 + 10?	13 + 13 + 13?

27. How many are—

4 + 6 + 7?	7 + 8 + 9?	10 + 11 + 12?	20 + 30 + 5?
5 + 8 + 9?	8 + 4 + 7?	11 + 12 + 13?	40 + 30 + 6?
6 + 7 + 12?	9 + 8 + 6?	14 + 13 + 12?	50 + 30 + 8?

28. A farmer bought a cow for 35 dollars, and a wagon for 42 dollars; what did both cow and wagon cost?

29. A lady paid 45 dollars for a coat, and 18 dollars for a dress; what did she pay for both?

30. A boy paid 55 dollars for a bicycle, and 24 dollars for a watch: what was the cost of both?

31. An arithmetic costs 65 cents, and a geography 75 cents; what is the cost of both?

32. A horse cost 175 dollars, and a cow 25 dollars; required the cost of both.

33. Count by 2's from 2 to 40.   Thus, 2, 4, 6, 8, etc.

34. Count by 2's from 1 to 41.

35. Count by 5's from 5 to 100.

36. Count by 10's from 10 to 100.

37. Count by 3's from 3 to 48; from 1 to 31.

38. Count by 4's from 4 to 60; from 1 to 45.

39. Count by 6's from 6 to 48; from 1 to 25.

II. SUBTRACTION.

1. James had 22 marbles, and lost 9 of them; how many had he remaining?

SOLUTION.—If James had 22 marbles, and lost 9 of them, he had remaining the difference between 22 marbles and 9 marbles, which is 13 marbles.

2. Anna had 18 dollars, and spent 7 of them; how many dollars had she remaining?

3. A farmer bought a wagon for 20 dollars, and sold it for 28 dollars; how much did he gain?

4. John had 26 apples, and gave 8 of them away; how many apples had he remaining?

5. Mr. Noble paid 32 dollars for a cow, and sold her for 25 dollars; how much did he lose?

6. A watch and a chain cost 45 dollars; what was the cost of the watch, if the chain cost 9 dollars?

7. John earns 28 dollars a week, and pays 6 dollars a week for his board; how much does he save each week?

8. May is 25 years old, and her brother is 8 years younger; required the age of her brother.

9. Henry had 37 cents, and spent 12 cents for candy; how much had he remaining?

10. Charles bought a book for 25 cents, and sold it for 35 cents; how much did he gain?

11. William had 27 hens, and sold 8 of them; how many had he remaining?

12. How many are—

40 − 30?	90 − 50?	75 − 10?	47 − 20?	57 − 40?
50 − 20?	80 − 30?	84 − 20?	62 − 30?	37 − 20?
60 − 40?	70 − 10?	76 − 30?	71 − 20?	43 − 30?

13. How many are—

48 − 6?	67 − 5?	36 − 12?	62 − 21?	36 − 14?
57 − 5?	29 − 7?	45 − 13?	33 − 13?	49 − 15?
49 − 7?	48 − 5?	56 − 12?	65 − 25?	57 − 25?

14. How many are—

22 − 9?	33 − 9?	25 − 17?	65 − 18?	62 − 23?
33 − 8?	42 − 7?	36 − 18?	47 − 22?	67 − 29?
43 − 5?	53 − 6?	43 − 25?	63 − 17?	73 − 36?

SUGGESTION.—A practical method of subtracting numbers like the above is as follows: Subtract 17 from 25. 17 and 3 are 20, and 5 are 25. 5 and 3 are 8.

15. How many are 40 men less 12 men?

16. How many are 62 miles less 20 miles?

17. How many are 50 bushels less 18 bushels?

18. How many are 43 days less 22 days?

19. How many are 65 weeks less 42 weeks?

20. How many are 84 yards less 17 yards?

21. How many are 75 dollars less 35 dollars?

22. How many are 62 feet less 27 feet?

23. How many are—

400 − 300?	450 − 150?	570 − 250?	840 − 510?
600 − 100?	640 − 120?	680 − 240?	930 − 720?
800 − 500?	760 − 160?	750 − 130?	670 − 230?

24. A line 90 feet long was cut into two unequal pieces; what was the length of the shorter piece if the longer part was 48 feet?

25. A gold watch cost 128 dollars, and the chain 42 dollars; what is the difference in their cost?

26. A drover bought 42 cows, and sold 25 of them; how many remained?

27. A farmer sold 12 acres of land from a farm containing 32 acres; how many acres had he remaining?

28. A tree 45 feet high was injured in a storm; how much remained if 22 feet were broken off?

29. James bought a bicycle for 45 dollars, and sold it for 32 dollars; how much did he lose?

30. The sum of two numbers is 39, and the smaller number is 16; what is the greater?

31. Count by 2's backward from 40 to 2; from 41 to 1.

32. Count by 5's backward from 50 to 5; from 100 to 50.

33. Count by 3's backward from 48 to 3; from 49 to 1.

34. Count by 4's backward from 48 to 4; from 50 to 2.

35. Count by 6's backward from 54 to 6; from 55 to 1.

III. ADDITION AND SUBTRACTION.

1. William had 12 cents, and earned 15 cents more: he then spent 16 cents; how much money had he remaining?

2. Ross had 25 cents: he gave 5 cents for a top, and 12 cents for a ball; how much had he remaining?

3. A farmer having 32 sheep sold 12 of them to A, and 15 to B; how many had he remaining?

4. A boy bought a sled for 40 cents, and a wagon for 35 cents; how much will he gain if he sells them both for 90 cents?

5. Mr. White paid 28 dollars for a cow, and at the end of two weeks sold her for 40 dollars; how much did he gain if her feed cost him 2 dollars?

6. Charles had 28 cents, of which he spent 15 cents, and afterward earned 22 cents; how much money did he then have?

7. A man bought a cow for 36 dollars, and a calf for 12 dollars; how much did he gain if he sold them both for 50 dollars?

8. William paid 25 cents for a basket of fruit, and 18 cents for a melon; how much should he receive for both to gain 10 cents?

9. A coat, vest, and pair of trousers cost 30 dollars; what was the cost of the trousers if the vest cost 5 dollars, and the coat 18 dollars?

10. John bought a wagon for 40 dollars; he hired it out for 12 dollars, and afterward sold it for 35 dollars; what was his gain by the transaction?

What is the value—

11. Of $5 + 8 - 3$?

12. Of $10 - 2 + 12$?

13. Of $18 + 7 - 6$?

14. Of $22 + 12 - 8$?

15. Of $32 + 13 - 5$?

16. Of $42 - 13 + 8$?

What is the value—

17. Of 36 − 9 + 14?
18. Of 45 − 7 + 9?·
19. Of 50 + 8 − 17?
20. Of 43 + 8 − 12?
21. Of 24 − 9 + 12?
22. Of 36 − 4 + 13?
23. Of 28 + 12 − 10?

24. Of 49 + 1 − 16?
25. Of 62 − 14 − 5?
26. Of 19 − 8 + 30?
27. Of 47 + 23 − 16?
28. Of 32 + 28 − 30?
29. Of 60 + 40 − 20?
30. Of 100 − 30 + 15?

31. A sold a horse for 75 dollars, and thereby gained 10 dollars; how much would he have gained if he had sold him for 87 dollars?

32. A drover having 100 sheep sold 37 of them, and bought 9 more; how many sheep had he then?

33. What number is as much more than 30 as 18 is less than 30?

34. A man having 40 dollars spent 15 of them, and afterward earned 22 dollars; how much money did he then have?

35. A bought a sheep for 12 dollars, and a cow for 32 dollars; how much would he gain by selling them both for 60 dollars?

36. Mr. B sold a bicycle for 75 dollars, and thereby lost 12 dollars; how much would he have lost if he had sold it for 50 dollars?

37. Three men built 60 rods of wall, of which the first built 16 rods, and the second 28 rods; how many rods did the third man build?

38. Wilson bought a knife for 45 cents, and sold it for 70 cents; he then bought a sled for 55 cents, and sold it for 67 cents; how much did he gain on both?

39. What number is as much greater than 50 as 31 is less than 42?

40. Wesley having a certain number of cents lost 10, spent 5. earned 25, and then had 50 cents; how much had he at first?

IV. MULTIPLICATION.

1. If a yard of gingham costs 15 cents, what will 12 yards cost?

SOLUTION.—If 1 yard of gingham costs 15 cents, 12 yards will cost 12 times 15 cents, which are 180 cents. Therefore, etc.

2. What will 12 balls cost, at 10 cents apiece? 1,2 0
3. What will 12 melons cost, at 11 cents apiece?'
4. What will 12 sheep cost, at 12 dollars apiece?
5. What will 12 pigs cost, at 13 dollars apiece?
6. What will 12 shad cost, at 16 cents apiece?

7. What will 12 hens cost, at 40 cents apiece?

8. What will 13 turkeys cost, at 80 cents apiece?

9. What will 15 cows cost, at 30 dollars apiece?

10. If a quart of berries costs 15 cents, what will 8 quarts cost?

11. At 18 cents a dozen, what will 8 dozen eggs cost?

12. If 10 men can do a piece of work in 16 days, how long will it take 1 man to do it?

13. A farmer bought 9 horses, at 80 dollars a head; how much did they cost him?

14. Helen bought 11 yards of ribbon, at 22 cents a yard; how much did it cost her?

15. John can ride 40 miles a day on his bicycle; how far can he ride in 12 days?

16. A train goes 35 miles an hour; how far will it go in 12 hours?

17. What will 9 acres of land cost, at 105 dollars an acre?

18. If 15 men can do a piece of work in 20 days, how long will it take 1 man to do it?

19. B bought 12 pairs of chickens, at 50 cents a pair; how much did they cost?

20. How many are—

11×1?	11×4?	11×7?	11×10?
11×2?	11×5?	11×8?	11×11?
11×3?	11×6?	11×9?	11×12?

21. How many are—

12×1?	12×4?	12×7?	12×10?
12×2?	12×5?	12×8?	12×11?
12×3?	12×6?	12×9?	12×12?

22. How many are—

13×1?	13×4?	13×7?	13×10?
13×2?	13×5?	13×8?	13×11?
13×3?	13×6?	13×9?	13×12?

Find the value—

23. Of 8 loaves of bread, at 14 cents a loaf.

24. Of 10 tons of hay, at 18 dollars a ton.

25. Of 13 sheep, at 9 dollars a head.

26. Of 22 pairs of boots, at 6 dollars a pair.

27. Of 9 cows, at 45 dollars a head.

28. Of 12 watches, at 40 dollars apiece.

Find the value—

29. Of 15 bushels of wheat, at 50 cents a bushel.

. *30.* Of 12 bushels of corn, at 40 cents a bushel.

31. Of 9 bushels of potatoes, at 80 cents a bushel.

32. Of 8 yards of dress goods, at 32 cents a yard.

33. Of 30 yards of cloth, at 12 cents a yard.

34. There are 16 ounces in a pound avoirdupois; how many ounces in 8 pounds? In 9 pounds? In 10 pounds? In 12 pounds?

35. There are 12 inches in a foot; how many inches in 13 feet? In 15 feet? In 20 feet? In 25 feet?

36. There are 7 days in a week; how many days are there in 12 weeks? In 30 weeks? In 52 weeks? In 60 weeks? In 71 weeks?

37. There are 12 months in a year; how many months in 10 years? In 12 years? In 15 years? In 25 years? In 30 years?

38. There are 24 hours in a day; how many hours in 5 days? In 6 days? In 7 days? In 10 days?

39. There are 63 gallons in a hogshead; how many gallons in 5 hogsheads? In 6 hogsheads? In 7 hogsheads? In 10 hogsheads?

40. There are 60 minutes in an hour; how many minutes in 8 hours? In 12 hours? In 15 hours?

41. How many sheets of paper in 8 quires, if there are 24 sheets in one quire?

42. How many are—

2×25?	5×25?	2×50?	5×50?
3×25?	6×25?	3×50?	6×50?
4×25?	7×25?	4×50?	7×50?

43. How many are—

5×20?	8×25?	8×60?	4×125?
4×30?	7×30?	9×70?	5×120?
6×40?	9×25?	12×75?	6×150?

44. A earns 20 dollars a week, and pays 6 dollars a week for his board; how much can he save in 8 weeks? In 10 weeks?

45. A farmer sold 8 dozen eggs, at 20 cents a dozen, and 10 pounds of butter, at 22 cents a pound; how much did he receive for both?

46. William bought 8 melons, at 10 cents each, and 2 dozen oranges, at 30 cents a dozen; how much would he gain by selling all of them for 150 cents?

47. A and B start from the same place, and travel in the same direction,—A at the rate of 15 miles an hour, and B at the rate of 9 miles an hour; how far apart will they be in 10 hours? In 12 hours? In 20 hours?

48. C and D start from the same place, and travel in opposite directions,—C at the rate of 8 miles an hour, and D at the rate of 12 miles an hour; how far apart will they be in 12 hours? In 15 hours?

49. A merchant bought 25 plows, at 10 dollars apiece; he sold 10 of them at 12 dollars each, and the remainder at 15 dollars each; how much did he gain?

50. A farmer bought 30 sheep, at 8 dollars a head; he sold 12 of them at 10 dollars apiece, and the remainder at 7 dollars apiece; did he gain or lose, and how much?

V. DIVISION.

1. At 5 cents apiece, how many oranges can be bought for 80 cents?

SOLUTION.—If 1 orange costs 5 cents, for 80 cents you can buy as many oranges as 5 is contained times in 80, which are 16. Therefore, etc.

2. If 6 boys earn 96 cents, how much does one boy earn?

SOLUTION.—If 6 boys earn 96 cents, 1 boy earns one sixth of 96 cents, or 16 cents.

NOTE.—These two solutions indicate the best practical methods of solving the two classes of problems that may arise in division.

3. John rode 75 miles in 5 days; what was the average number of miles for each day?

4. There are 8 quarts in a peck; how many pecks in 240 quarts?

5. Eight acres of land were sold for 960 dollars; what was the average price per acre?

6. A man can do a piece of work in 279 days; in what time can 9 men do it?

7. A boy paid 64 cents for oranges, at 4 cents apiece; how many did he buy?

8. An express train ran 210 miles in 7 hours; what was the average speed per hour?

9. A merchant bought 22 barrels of flour for 88 dollars; what was the price per barrel?

10. A school building of 11 rooms contains 550 desks; what is the average number of desks in each room?

11. Eight men can build 320 rods of fence in a month; how many rods can 1 man build in the same time?

12. If 12 suits of clothes cost 156 dollars, what will 1 suit cost?

13. James paid 728 dollars for 7 horses; what was the average price paid per head?

14. A bought 20 cows for 500 dollars; what was the average price per head?

15. Thirteen boxes contain 1326 pieces of soap; what is the average number of pieces in a box?

16. How many 4-quart cans can be filled from a cask containing 480 quarts?

17. If 15 melons cost 225 cents, what will 1 melon cost?

18. If 16 pounds of butter cost 256 cents, what will 1 pound cost?

19. If 12 gallons of oil cost 168 cents, what will 1 gallon cost?

20. If 20 yards of dress goods cost 420 cents, what will 1 yard cost?

21. How many times is 20 contained—

In 60?	In 140?	In 160?	In 340?
In 80?	In 180?	In 120?	In 400?
In 100?	In 200?	In 320?	In 560?

22. How many times is 12 contained—

In 48?	In 84?	In 120?	In 156?
In 60?	In 96?	In 132?	In 168?
In 72?	In 108?	In 144?	In 180?

23. How many times is 10 contained—

In 70?	In 100?	In 140?	In 170?
In 80?	In 120?	In 150?	In 180?
In 90?	In 130?	In 160?	In 200?

24. B bought 12 barrels of sugar for 156 dollars; what was the cost per barrel?

25. Eleven men earned 154 dollars in a week; how much did 1 man earn in the same time?

26. Six acres of land cost 726 dollars; what was the cost of 1 acre?

Required the price per pound, yard, etc.—

27. If 8 pounds of coffee cost 248 cents.

28. If 12 yards of cloth cost 132 dollars.

29. If 15 gallons of oil cost 450 cents.

30. If 25 acres of land cost 2500 dollars.

31. If 9 yards of dress goods cost 198 cents.

32. If 10 rods of fence cost 560 cents.

33. If 16 pounds of beef cost 256 cents.

34. If 15 pounds of lard cost 120 cents.

35. In 18 there are how many 4's?

SOLUTION.—In 18 there are *four* 4's, and a remainder of 2.

36. In 22, how many 5's? 6's? 7's?

37. In 42, how many 8's? 9's? 10's?

38. In 45, how many 7's? 8's? 10's?

39. In 56, how many 9's? 10's? 11's?

40. In 75, how many 7's? 8's? 9's?

41. In 80, how many 8's? 10's? 20's?

42. In 100, how many 20's? 25's? 50's?

43. In 120, how many 10's? 12's? 15's?

44. In 144, how many 8's? 9's? 12's? 16's?

45. In 150, how many 6's? 15's? 25's? 50's?

VI. MULTIPLICATION AND DIVISION.

1. 12 times 3 are how many times 9?

2. 6 times 10 are how many times 12?

3. 8 times 12 are how many times 4?

4. 9 times 8 are how many times 12?

5. 8 times 10 are how many times 4?

6. 10 times 12 are how many times 6?

7. 6 times 16 are how many times 3? 8?

8. 5 times 20 are how many times 10? 25?

9. 12 times 9 are how many times 3? 4? 6?

10. 6 times 15 are how many times 3? 5? 10?

11. 4 times 25 are how many times 5? 10? 20?

12. 11 times 12 are how many times 3? 4? 6?

13. 7 times 18 are how many times 3? 6? 9?

14. 8 times 21 are how many times 4? 7? 42?

15. 12 times 25 are how many times 6? -15? 30?

16. How many pounds of sugar, at 6 cents a pound, can be bought with 4 pounds of butter, at 24 cents a pound?

17. If 9 men can do a piece of work in 10 days, how long will it require 6 men to do the same work?

18. How many sheep, at 6 dollars a head, must be given for 5 cows, at 30 dollars a head?

19. Ten men can do a piece of work in 40 days; how long will it take 8 men to do the same work?

20. A sold his sheep for 720 dollars, being at the rate of 8 dollars a head; how much more would he have received if he had sold them for 10 dollars a head?

21. What number multiplied by 6 gives a product of 16 less than 100?

22. B bought 150 dollars' worth of coal, at 5 dollars a ton, and sold it at 4 dollars a ton; how much did he lose?

23. The product of three numbers is 105; what is the third number if the other two are 3 and 5?

24. A man receives 15 cents an hour, and his son 8 cents an hour; how much will they both receive for a day's work of 10 hours?

25. A bought 126 dollars' worth of sheep, at 6 dollars a head, and sold them at 10 dollars a head; how much did he gain?

26. A farmer bought a number of cows at an average price of 20 dollars a head, and sold them at 24 dollars a head,—thereby gaining 116 dollars; how many cows did he buy?

27. A earns 15 dollars a week, and pays 5 dollars a week for his board; in how many weeks can he save 100 dollars?

28. A merchant bought 12 lamps for 360 cents; how much would he gain on each lamp by selling them at 50 cents apiece?

29. A and B start from two cities, 144 miles apart; in how many hours will they meet if A travels 5 miles an hour, and B 7 miles?

30. A farmer sold 9 pounds of butter at 16 cents a pound, and took in exchange calico at 12 cents a yard; how many yards did he get?

NOTE.—The sign ( ), called the *Parentheses*, is used to indicate that the numbers inclosed by the curved lines are all subject to the same operation.

Thus, $(9 - 2 + 4) \times 6$ indicates that $9 - 2 + 4$, or 11, are to be multiplied by 6.

The *Vinculum*, $\overline{}$, is sometimes used instead of the parentheses or in connection with them.

Thus, $\overline{9 - 2} + 4 \times 6$ is the same as $(9 - 2 + 4) \times 6$.

In problems in which +, −, and × or ÷ occur, the multiplication or division should be performed *before* the additions or subtractions are made.

Thus, $4 + 3 \times 5 - 6 + 2 = 4 + 15 - 3 = 16$.

Problems in which × and ÷ occur in succession, admit of two meanings.

Thus, $12 \div 2 \times 3$ is either $6 \times 3 = 18$, or $12 \div 6 = 2$.

It is necessary in such cases to use the parentheses or the vinculum to indicate the meaning.

Thus, $(12 \div 2) \times 3 = 18$, or $12 \div (2 \times 3) = 2$.

What is the value—

31. Of $(4 \times 3) \div 2$?	*40.* Of $(10 + 2) \times 6$?
32. Of $4 \times (8 \div 4)$?	*41.* Of $(18 \div 6) \times 7$?
33. Of $5 \times (12 \div 3)$?	*42.* Of $(24 \div 8) \times 7$?
34. Of $(18 \div 6) \times 5$?	*43.* Of $5 \times (14 \div 2)$?
35. Of $(21 \div 7) \times 4$?	*44.* Of $6 \times (27 \div 9)$?
36. Of $(30 \div 6) \times 3$?	*45.* Of $7 \times (32 \div 8)$?
37. Of $(6 \times 4) \div 3$?	*46.* Of $(20 \div 4) \times 8$?
38. Of $(8 \div 2) \times 6$?	*47.* Of $(36 \div 6) \times 9 - 4$?
39. Of $(9 \div 3) \times 8$?	*48.* Of $(81 \div 9) \times 4 + 5$?

49. If 8 oranges cost 32 cents, what will 10 oranges cost?

SOLUTION.—If 8 oranges cost 32 cents, 1 orange will cost one eighth of 32 cents, or 4 cents, and 10 oranges will cost 10 times 4 cents, which are 40 cents. Therefore, etc.

50. If 9 sheep cost 54 dollars. what will 15 sheep cost?

51. If 12 hats cost 36 dollars, what will 20 hats cost?

52. If 16 cows cost 480 dollars, what will 30 cows cost?

53. If 10 melons cost 90 cents, what will 40 melons cost?

54. If 15 plows cost 120 dollars, what will 20 plows cost?

55. If 30 apples cost 90 cents, what will 50 apples cost?

56. If 5 wagons cost 200 dollars, what will 12 wagons cost?

57. If 10 bicycles cost 500 dollars, what will 15 bicycles cost?

58. If 12 coats cost 96 dollars, what will 10 coats cost?

59. If 20 horses cost 3000 dollars, what will 3 horses cost?

60. If 12 melons cost 48 cents, what will 5 melons cost?

61. If 13 stamps cost 26 cents, what will 16 stamps cost?

62. If 11 books cost 33 dollars, what will 15 books cost?

63. If 9 bananas cost 18 cents, what will 11 bananas cost?

64. If 10 brooms cost 180 cents, what will 6 brooms cost?

III. UNITED STATES MONEY.

United States Money is the legal currency of the United States.

TABLE.

10 mills	=	1 cent, ct.
10 cents	=	1 dime, d.
10 dimes	=	1 dollar, $.
10 dollars	=	1 eagle, E.

The denominations generally used in business transactions are *dollars* and *cents*. These are written with a (.), called a *decimal point*, between them; thus, $25.75 is read *25 dollars and 75 cents.*

The coins of the United States now current are:

Gold—the *quarter-eagle, half-eagle, eagle,* and *double eagle.*
Silver—the *dime, quarter-dollar, half-dollar,* and *dollar.*
Nickel—the *five-cent piece.*
Bronze—the *cent.*

1. How many mills in 3 cents? In 8 cents? In 12 cents? In 15 cents?

2. How many cents in 5 dimes? In 8 dimes? In 16 dimes? In 80 mills? In 100 mills? In 130 mills?

3. How many dimes in 6 dollars? In $12? In $18? In 150 cents? In 500 cents? In 200 mills? In 800 mills?

4. How many dollars in 10 eagles? In 40 dimes? In 70 dimes? In 600 cts. ? In 1000 cts. ? In 40 eagles?

5. How many cents in $5.24? In $7.85? In $12.25?

6. A bought a pair of boots for $6.50, and a hat for $3.25; what was the cost of both?

7. Ross paid $1.25 for a whip, $8.25 for a robe, and $5.50 for a blanket; how much did all cost?

8. A sled cost $1.20, a wagon $2.40, and a coat $3.60; how much did all cost?

9. Frank paid $2.40 for a book, and $1.60 for a hat; how much more did he pay for the book than for the hat?

10. Mary paid $6.75 for a cloak, and $5.25 for a dress; how much more did she pay for the cloak than for the dress?

11. C bought a cow for $25.25, and sold her for $30.75; how much did he gain?

12. A horse was bought for $125.50, and sold for $150; required the gain.

13. What will 10 sheep cost, at $6.25 a head?

14. What will 5 tons of coal cost, at $5.25 a ton?

15. What will 6 bicycles cost, at $55 apiece?

16. If 5 tons of hay cost $90.25, what will 1 ton cost?

17. How much are 5 times 30 cents? 10 times 20 cents? 8 times 25 cents? 6 times 50 cents?

18. Divide $1.50 by 5. $2.00 by 8. $3.00 by 4. $5.00 by 25.

19. How much is gained on 8 books, bought at $1.05 and sold at $1.25 each?

20. If eggs are bought at 25 cents a dozen, and sold at 3 cents apiece, what is gained per dozen?

21. How many sheep, at $8 a head, can be bought for $104?

22. What is the cost of 15 yards of cloth, at $2.05 a yard?

23. What is the entire cost of 12 pounds of sugar, at 6 cents a pound, and 8 gallons of oil, at 20 cents a gallon?

24. A bought 15 plows for $90, and sold them at $6.50 apiece; required his gain.

25. If 6 sheep cost $36.60, how much is that apiece?

26. A hatter bought caps at $6 a dozen, and sold them at 75 cts. apiece; how much did he gain on each cap?

27. A bought 8 cows for $320, and sold them at the rate of 2 cows for $84.50; how much did he gain on each cow?

28. A merchant bought 4 dozen eggs, at 9 cents a dozen, and sold them at a cent apiece; how much was his profit?

29. I went to the store with $12.25; I bought a dress for $5.25, a hat for $3.25, and an umbrella for $1.25; how much money had I left?

30. If a house cost $1200, and a barn one sixth as much, how much did both cost?

31. If a gold chain cost $25.50, and a watch 4 times as much, how many dollars did both cost?

Making Change.

1. If you buy 11 pounds of sugar, at 6 cents a pound, and give your grocer a dollar-bill in payment, how should he count the change due you?

Answer.—Giving me the sugar, he should count that at 11 × 6 cts., which are 66 cents; and then hand me in succession 4 cents, a 5-cent piece, and a quarter-dollar, counting **66, 70, 75, $1.** This is the most convenient way to make change.

NOTE.—When not specified, any denominations in current use may be used. In the following examples use the *smallest possible number* of coins and bills in making change:

2. How should the change be counted, if you buy 5 pounds of coffee at 32 cts. a pound, and give in payment a 5-dollar bill?

3. How should the change be counted in taking $3.05 out of $5?

4. How should the change be counted in taking $1.65 out of $10?

How should the change be counted in—

5. Taking 39 cts. out of $2?   Taking $1.16 out of $2?

6. Taking $5.47 out of $10?   Taking $16.32 out of $20?

7. Taking $3.43 out of $5?   Taking $7.19 out of $10?

8. Taking 13 cts. out of a half-dollar?   Taking 39 cts. out of a dollar?

9. Taking $1.25 out of a quarter-eagle?

10. Taking $18.65 out of a double eagle?

Count the correct change in each of the following cases:

11. 12 pencils at 4 cts. apiece, out of $2.

12. 15 brushes at 11 cts. apiece, out of $5.

13. 11 melons at 8 cts. apiece, out of $2.

14. 12 books at 22 cts. apiece, out of $10.

15. 10 baskets at 42 cts. apiece, out of $10.

16. 20 collars at 18 cts. apiece, out of $5.

17. 9 caps at 50 cts. apiece, out of $10.

18. 8 clocks at $6.05 apiece, out of $50.

19. 15 bottles at 30 cts. apiece, out of $10.

20. 30 books at 25 cts. apiece, out of $20.

21. 6 collars at 13 cts. apiece, out of $1.

22. 13 loaves of bread at 7 cts. apiece, out of a half-eagle.

23. 14 dishes at 40 cts. apiece, out of a gold eagle.

IV. FACTORS, DIVISORS, AND MULTIPLES.

I. FACTORS.

1. WHAT two numbers multiplied together will produce 6? 8? 10? 14? 15? 21? 35?

2. What two numbers multiplied together will produce 55? 65? 77? 91? 51? 39? 57?

The numbers which when multiplied together produce the given number are called the **Factors** of that number.

NOTE.—*One* and *the number itself* are not considered as factors.

3. What two factors will produce 12? 16? 20? 30? 32? 40? 45? 48? 50?

4. What two factors will produce 22? 33? 44? 39? 51? 65? 85?

5. What two factors will produce 15? 20? 25? 30? 35? 40? 45? 50?

6. What three factors will produce 12? 18? 30? 42? 70? 105? 66? 110?

7. What four factors will produce 16? 24? 36? 40? 54?

8. What are the factors of 23?

A number that has no factors is called a *Prime Number;* all other numbers are called *Composite Numbers.*

9. Name the prime numbers between 1 and 20. Between 20 and 50.

10. Name the composite numbers between 50 and 60. Between 60 and 70.

11. What numbers between 1 and 25 are exactly divisible by 2? Which are not?

A number exactly divisible by 2 is an *Even Number.* A number that is not exactly divisible by 2 is an *Odd Number.*

12. What prime numbers multiplied together will produce 35? 55? 33? 49?

3   33

13. What prime numbers multiplied together will produce 42? 56? 63? 75? 99?

When the factors of a number are prime numbers, they are called *Prime Factors*.

· *14.* What are the prime factors of 24? 28? 36? 42? 45? 52? 54?

15. What are the prime factors of 80? 81? 85? 87? 91? 93? 96? 99?

16. What numbers between 20 and 50 have 3 for a factor?

17. What numbers between 60 and 100 have 5 for a factor?

II. DIVISORS.

Since 6 exactly divides 12, it is therefore called a **Divisor** of 12.

Name the divisors—

1. Of 24; 45.	*4.* Of 36; 63.
2. Of 35; 49.	*5.* Of 45; 49.
3. Of 33; 55.	*6.* Of 50; 75.

Since 6 exactly divides both 12 and 24, it is therefore called a *Common Divisor* of 12 and 24.

Name the common divisors—

7. Of 9 and 12.	*10.* Of 40 and 56.
8. Of 12 and 18.	*11.* Of 60 and 72.
9. Of 24 and 36.	*12.* Of 72 and 96.

Since 12 is the largest number that will exactly divide 12 and 24, it is therefore called the *Greatest Common Divisor* of 12 and 24.

Find the greatest common divisor—

13. Of 6 and 15.	*22.* Of 56 and 84.
14. Of 18 and 36.	*23.* Of 45 and 75.
15. Of 9 and 24.	*24.* Of 68 and 102.
16. Of 16 and 28.	*25.* Of 108 and 144.
17. Of 33 and 77.	*26.* Of 90 and 105.
18. Of 42 and 48.	*27.* Of 210 and 240.
19. Of 26 and 39.	*28.* Of 360 and 450.
20. Of 24 and 60.	*29.* Of 120 and 150.
21. Of 44 and 99.	*30.* Of 480 and 560.

III. MULTIPLES.

Since 12 is exactly divisible by 6, it is therefore called a **Multiple** of 6.

Find a multiple of each of the following numbers:

1.	3.	5.	6.	*4.* 16.	20.	25.
2.	4.	8.	12.	*5.* 30.	40.	45.
3.	10.	9.	15.	*6.* 34.	36.	31.

Since 12 is exactly divisible by both 4 and 6, it is therefore called a *Common Multiple* of 4 and 6.

Find a common multiple of—

      7. Both 6 and 12.   *11.* Both 9 and 24.
      8. Both 8 and 10.   *12.* Both 15 and 18.
      9. Both 9 and 12.   *13.* Both 16 and 20.
      10. Both 10 and 15.   *14.* Both 18 and 24.

Since 18 is the least number that is exactly divisible by both 6 and 9, it is therefore called their *Least Common Multiple.*

Find the least common multiple—

      15. Of 12 and 18.   *20.* Of 30 and 40.
      16. Of 15 and 20.   *21.* Of 20 and 45.
      17. Of 20 and 30.   *22.* Of 60 and 90.
      18. Of 21 and 63.   *23.* Of 60 and 80.
      19. Of 20 and 25.   *24.* Of 56 and 70.

Find the least common multiple—

      25. Of 2, 3, and 4.   *29.* Of 3, 5, and 8.
      26. Of 3, 4, and 6.   *30.* Of 4, 6, and 8.
      27. Of 5, 7, and 10.   *31.* Of 3, 7, and 5.
      28. Of 4, 5, and 6.   *32.* Of 5, 7, and 8.

33. What is the least sum of money with which a boy can purchase either apples, at 3 cents each, or oranges, at 5 cents each?
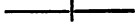
34. What is the least quantity of water that can be exactly measured by vessels which hold either 3, 4, or 5 gallons each?

35. What is the least number of acres that can be exactly divided into fields of 4, 5, or 8 acres each?

V. FRACTIONS.

I. Ideas of Fractions.

HALF.

1. **If** a line is divided into two equal parts, what is one of these parts called? What are two of these parts called? How many halves in a whole line?

2. What is 1 half of 4?

SOLUTION.—One half of 4 is 4 divided by 2, which is 2.

3. What is 1 half of 6? Of 8? Of 10? Of 12?

4. What is 1 half of 18? Of 24? Of 36? Of 40?

5. What is 1 half of 60? Of 66? Of 80? Of 100?

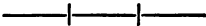*6.* If a melon costs 8 cents, what will 1 half of a melon cost?

7. If a pound of candy costs 18 cents, what will 1 half of a pound cost?

8. If 1 half of a pound of sugar costs 10 cents, what will a pound cost?

9. If 1 half of an acre of land is worth $40, what is an acre worth?

10. What is 1 half of 5? Of 7? Of 9? Of 13?

THIRD.

1. If a line is divided into three equal parts, what is one of these parts called? What are two of these parts called? What are three of these parts called?

2. How many thirds in a whole line? How many thirds in an apple? In an orange?

3. What is 1 third of 6? Of 9? Of 12?

4. What is 1 third of 15? Of 18? Of 21?

5. What is 1 third of 24? Of 75? Of 90?

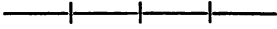*6.* What is 1 third of 270? Of 360? Of 450?

7. If a pound of butter costs 36 cents, what will 1 third of a pound cost?

8. There are 12 inches in a foot; how many inches in 1 third of a foot?

9. Anna had 30 apples, and gave 1 third of them to May; how many did she give to May?

10. There are 30 days in a month; how many days in 1 third of a month?

11. What are 2 thirds of 12?

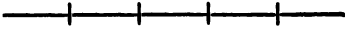SOLUTION.—1 third of 12 is 4; and 2 thirds of 12 are 2 times 4, which are 8.

12. What are 2 thirds of 15?   Of 18?   Of 24?

13. What are 2 thirds of 42?   Of 60?   Of 72?

14. What are 2 thirds of 90?   Of 120?   Of 210?

15. What are 2 thirds of 45?   Of 66?   Of 48?

16. If a yard of cloth costs $9, what will 2 thirds of a yard cost?

17. If a pound of candy costs 21 cents, what will 2 thirds of a pound cost?

FOURTH.

1. If a line is divided into four equal parts, what is one of these parts called? What are two of these parts called?   What are three of these parts called?

2. How many fourths in a whole line?   How many fourths in an apple?

3. What is 1 fourth of 8?   Of 12?   Of 20?

4. What is 1 fourth of 16?   Of 24?   Of 36?

5. What is 1 fourth of 40?   Of 60?   Of 72?

6. What are 2 fourths of 8?   Of 12?   Of 16?

7. What are 2 fourths of 20?   Of 24?   Of 32?

8. What are 3 fourths of 12?   Of 20?   Of 28?

9. What are 3 fourths of 40?   Of 48?   Of 100?

10. If a box of candy costs 12 cents, what will 1 fourth of a box cost?   3 fourths of a box?

11. A sold a cow for $40, and gained 1 fourth of this amount; how much did he gain?

12. C sold a carriage for $64, and gained 1 fourth of this amount; how much did he gain?

13. There are 60 minutes in an hour; how many minutes in 1 fourth of an hour?   In 1 half of an hour?   In 3 fourths of an hour?

FIFTH.

1. If a line is divided into five
equal parts, what is one of these
parts called? What are two of these parts called? Three of these
parts? Four of these parts?

2. How many fifths in a line?

3. How many fifths in an orange?

4. How many fifths in 2 oranges? In 3 oranges?

5. What is 1 fifth of 10? Of 15? Of 20?

6. What is 1 fifth of 25? Of 30? Of 40?

7. What are 2 fifths of 10? Of 20? Of 30?

8. What are 2 fifths of 15? Of 25? Of 35?

9. What are 3 fifths of 20? Of 35? Of 40?

10. What are 3 fifths of 30? Of 45? Of 50?

11. What are 4 fifths of 20? Of 35? Of 75?

12. What are 4 fifths of 25? Of 50? Of 100?

13. James earned $30, and spent 2 fifths of it; how much did he
spend? How much had he remaining?

14. A paid 4 fifths of $50 for a watch; required the cost of the
watch.

15. A farmer raised 100 bushels of wheat, and sold 3 fifths of it; how
many bushels did he sell? How many bushels had he remaining?

16. Martin owns a farm of 120 acres; if he should sell 2 fifths of it,
how many acres would he have remaining?

17. William had $60, and earned 3 fifths as much more; how much
did he then have?

SIXTH, SEVENTH, EIGHTH, NINTH, TENTH.

1. If an apple is divided into six equal parts, what is one of these
parts called? What are two of these parts called?

2. What is 1 sixth of 6? Of 12? Of 18?

3. What are 2 sixths of 24? Of 30? Of 36?

4. What are 3 sixths of 18? Of 42? Of 60?

5. What are 4 sixths of 48? Of 54? Of 72?

6. A horse cost $120, and a carriage 5 sixths as much; what did
the carriage cost?

7. A watch cost $60, and a chain 1 sixth as much; required the
cost of the chain.

8. Henry had $150 in bank, and paid 4 sixths of it for a bicycle; required the cost of the bicycle.

9. How many sevenths in a single thing?

10. What is 1 seventh of 14? Of 28? Of 35?

11. What are 2 sevenths of 21? Of 42? Of 49?

12. What are 3 sevenths of 14? Of 56? Of 63?

13. What are 4 sevenths of 70? Of 77? Of 84?

14. What are 5 sevenths of 35? Of 42? Of 63?

15. A farmer had 14 sheep, and a dog killed 1 seventh of them; how many remained?

16. A drover had 28 cows, and sold 3 sevenths of them; how many did he sell? How many remained?

17. A newsboy had 140 papers, and sold 5 sevenths of them; how many had he remaining?

18. How many eighths are there in one?

19. What is 1 eighth of 16? Of 24? Of 32?

20. What are 3 eighths of 40? Of 48? Of 56?

21. What are 5 eighths of 32? Of 64? Of 72?

22. What are 6 eighths of 48? Of 80? Of 88?

23. What are 7 eighths of 56? Of 72? Of 96?

24. Charles had 24 cents, and lost 3 eighths of them; how many cents had he remaining?

25. A has 48 sheep, and B has 7 eighths as many; how many sheep has B?

26. Horace earns $80 a month, and Ross earns 5 eighths as much; how much does Ross earn per month?

27. How many ninths are there in one?

28. What is 1 ninth of 18? Of 36? Of 54?

29. What are 2 ninths of 27? Of 45? Of 63?

30. What are 4 ninths of 72? Of 81? Of 90?

31. What are 5 ninths of 36? Of 54? Of 63?

32. What are 7 ninths of 99? Of 108? Of 144?

33. Mary had 27 pinks, and gave 2 ninths of them away; how many pinks had she remaining?

34. If there are 63 gallons in a hogshead, how many gallons in 5 ninths of a hogshead?

35. John paid $270 a year for his board, and 7 ninths as much for clothing; how much did both board and clothing cost?

36. What is 1 tenth of 20? Of 40? Of 50?

37. What are 3 tenths of 30? Of 60? Of 70?

38. What are 7 tenths of 50? Of 70? Of 100?

39. What are 9 tenths of 60? Of 80? Of 90?

40. What are 2 tenths of 120? Of 150? Of 160?

41. What are 4 tenths of 200? Of 300? Of 400?

42. Amos rode 40 miles, and walked back 7 tenths of this distance; how far was he then from the starting-place?

43. A kite was 60 yards from the ground, and fell 3 tenths of this distance; how far was it then from the ground?

44. A bought a horse for $210, and a cow for 2 tenths as much; how much did both cost?

II. DEFINITIONS OF FRACTIONS.

A **Fraction** is one or more of the equal parts of a unit.

A fraction is expressed by writing one number above another, with a short horizontal line between them. Thus, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{3}{4}$ are the forms used for *one half*, *one third*, and *three fourths*.

The **Denominator** is the number below the line, and shows into how many parts the unit is divided.

The **Numerator** is the number above the line, and shows how many parts are taken.

A **Proper Fraction** is one whose value is less than *one*, as $\frac{1}{2}$, $\frac{2}{3}$, etc.

An **Improper Fraction** is one whose value is equal to or greater than *one*, as $\frac{3}{3}$, $\frac{5}{3}$, etc.

A **Mixed Number** is the sum of a whole number and a fraction, as $4 + \frac{2}{3}$, written $4\frac{2}{3}$.

A fraction whose denominator is 10, 100, etc. is called a **Decimal Fraction.** Such fractions are usually expressed by placing a point (.), called the *decimal point*, before the numerator, and omitting the denominator. Thus, $\frac{3}{10} = .3$, $\frac{7}{10} = .7$, etc.

Read the following fractions, and tell the kind of each:

1. $\frac{2}{3}$.	*5.* $\frac{10}{100}$.	*9.* $6\frac{2}{3}$.	*13.* $\frac{101}{100}$.
2. $\frac{11}{2}$.	*6.* $\frac{23}{48}$.	*10.* $7\frac{1}{2}$.	*14.* $\frac{2}{11}$.
3. $\frac{20}{9}$.	*7.* $\frac{70}{12}$.	*11.* $\frac{81}{11}$.	*15.* $\frac{1}{7}$.
4. $\frac{5}{11}$.	*8.* $1\frac{1}{2}$.	*12.* $9\frac{7}{10}$.	*16.* $16\frac{4}{5}$.

III. Reduction of Fractions to Whole or Mixed Numbers.

1. How many ones in $\frac{12}{4}$?

SOLUTION.—In 1 there are $\frac{4}{4}$, and in $\frac{12}{4}$ there are as many *ones* as 4 is contained times in 12, which are 3. Therefore, etc.

2. How many ones in $\frac{8}{2}$? In $\frac{12}{2}$? In $\frac{30}{2}$? In $\frac{32}{2}$?

3. How many ones in $\frac{6}{3}$? In $\frac{12}{3}$? In $\frac{21}{3}$? In $\frac{24}{3}$?

4. How many ones in $\frac{16}{4}$? In $\frac{24}{4}$? In $\frac{36}{4}$? In $\frac{40}{4}$?

5. How many ones in $\frac{15}{5}$? In $\frac{35}{5}$? In $\frac{50}{5}$? In $\frac{60}{5}$?

6. Reduce $\frac{13}{3}$ to a mixed number.

7. In reducing $\frac{13}{3}$ to a mixed number, by what number do we divide the numerator 13?

8. How can we reduce an improper fraction to a whole or a mixed number without the analysis?

NOTE.—*To reduce an improper fraction to a whole or a mixed number, divide the numerator by the denominator.*

9. Reduce $\frac{14}{3}$ to a mixed number; $\frac{23}{3}$; $\frac{32}{7}$; $\frac{60}{7}$.

10. Reduce $\frac{17}{5}$ to a mixed number; $\frac{25}{8}$; $\frac{27}{8}$; $\frac{75}{8}$.

11. Reduce $\frac{20}{9}$ to a mixed number; $\frac{34}{9}$; $\frac{47}{9}$; $\frac{55}{9}$.

12. Reduce $\frac{28}{16}$ to a mixed number; $\frac{31}{16}$; $\frac{47}{16}$; $\frac{63}{16}$.

13. Reduce $\frac{40}{9}$ to a mixed number; $\frac{80}{9}$; $\frac{75}{8}$; $\frac{84}{4}$.

14. Reduce to mixed numbers $\frac{23}{11}$, $\frac{45}{12}$, $\frac{64}{7}$, and $\frac{87}{7}$.

15. Reduce to mixed numbers $\frac{41}{8}$, $\frac{63}{16}$, $\frac{57}{11}$, and $\frac{91}{12}$.

16. Reduce to mixed numbers $\frac{32}{7}$, $\frac{27}{8}$, $\frac{41}{9}$, and $\frac{50}{9}$.

17. Reduce to mixed numbers $\frac{70}{9}$, $\frac{73}{16}$, $\frac{85}{16}$, and $\frac{87}{16}$.

18. Reduce to mixed numbers $\frac{81}{9}$, $\frac{100}{9}$, $\frac{45}{16}$, and $\frac{80}{9}$.

19. Reduce to mixed numbers $\frac{99}{11}$, $\frac{120}{11}$, $\frac{100}{11}$, and $\frac{88}{11}$.

20. Reduce to mixed numbers $\frac{25}{9}$, $\frac{91}{8}$, $\frac{98}{8}$, and $\frac{100}{9}$.

21. If 4 pecks make a bushel, how many bushels are there in 17 pecks?

22. There are 12 inches in a foot; how many feet in 37 inches?

23. How many yards of cloth, at $6 a yard, can be bought for $26?

24. How many barrels of flour, at $4 a barrel, can be bought for $43?

25. How many pounds of sugar, at 6 cts. a pound, can be bought for 65 cents?

26. There are 4 quarts in a gallon; how many gallons in 47 quarts?

27. How long will it take to travel 70 miles, at the rate of 8 miles an hour?

28. How many· barrels of apples, at $5 a barrel, can be bought for $51 ?

29. A boy paid 17 half-dollars for a wagon; how many dollars did it cost him ?

30. A paid 29 quarter-dollars for some corn; how many dollars did it cost him ?

31. How many weeks, of 7 days each, in 73 days?

32. How many years, of 12 months each, in 95 months?

33. If you give to 46 beggars one tenth of a dollar each, how many dollars will it require ?

34. How many 7's in 78?   In 95?

35. How many 9's in 88?   In 100?

36. How many 12's in 123?   In 145?

37. At $6 a cord, how many cords of wood can you buy for $53?

38. At 10 cents a yard, how many yards of muslin can you buy for $1.17?

39. At 6 cents a quart, how many quarts of cherries can you buy for $1.21?

40. At 5 cents a quart, how many quarts of milk can you buy for $1.18?

IV. REDUCTION OF NUMBERS TO FRACTIONS.

1. How many halves in an apple?

2. How many halves in 5 apples?

SOLUTION.—In 1 apple there are 2 halves, and in 5 apples there are 5 times 2 halves, or 10 halves.   Therefore, etc.

3. How many thirds in 3?   In 5?   In 6?   In 7?

4. How many fourths in 4?   In 7?   In 8?   In 10?

5. How many fifths in 3?   In 4?   In 5?   In 6?

6. How many sixths in 5?   In 6?   In 7?   In 8?

7. How many sevenths in 8?   In 9?   In 10?   In 11?

8. How many tenths in 6?   In 7?   In 9?   In 10?

9. How many fifths in $3\frac{2}{5}$?

SOLUTION.—In 1 there are 5 fifths; in 3 there are 3 times 5 fifths, or 15 fifths; 15 fifths and 2 fifths are 17 fifths.   Therefore, in $3\frac{2}{5}$ there are $\frac{17}{5}$.

10. How many halves in 4½? In 6½? In 7½? In 9½?
11. How many thirds in 3½? In 4⅔? In 5½? In 8⅔?
12. How many fourths in 2¼? In 4¾? In 6¼? In 5¾?
13. How many fifths in 4⅗? In 6⅗? In 8⅖? In 7⅗?

14. In reducing a mixed number to an improper fraction, by what do we multiply the whole number? What do we add to the product?

NOTE.—*A mixed number may be reduced to an improper fraction by multiplying the whole number by the denominator of the fraction, adding the numerator to the product, and placing the result over the denominator.*

15. Reduce to improper fractions 4⅛; 6⅝; 8⅛; 9⅝.
16. Reduce to improper fractions 5⅘; 8½; 7¾; 9⅘.
17. Reduce to improper fractions 3⅛; 4⅝; 5⅝; 6⅞.
18. Reduce to improper fractions 4⅘; 5⅝; 7⅞; 8⅝.
19. Reduce to improper fractions 4₁₁; 6₁₁²; 7₁₁³; 8₁₁⁵.
20. Reduce to improper fractions 5₁₂¹; 7₁₀³; 8⅜; 9₁₁⁸.
21. Reduce to improper fractions 16½; 33½; 37¼; 62½.

22. A farmer gave some boys $4½ for planting corn; how many boys were there, if each boy received $½?

23. A gave 5½ bushels of corn to some poor persons; how many persons were there, if each person received ½ of a bushel?

24. A teacher gave his pupils $9¾; how many pupils were there, if each pupil received $¼?

25. A family uses ⅘ of a barrel of flour in a month; how long will 4⅘ barrels last them?

26. A man spent $10⅔ for books, at an average price of $⅓ each; how many books did he buy?

27. A contractor can build ₁₂¹ of a mile of railroad in a week; how long will it take him to build 4₁₂⁵ miles of road?

28. A gardener divided 2₁₁⁸ acres of land into small fields containing ₁₁¹ of an acre each; how many fields did he make?

29. Reduce to improper fractions 8₁₂⁵; 10₁₁¹; 4₂₀⁷; 12⅘.
30. Reduce to improper fractions 6₁₂⁵; 11₁₂⁵; 50⅝; 70¼.
31. Reduce to improper fractions 20⅘; 8₁₅⁷; 7⅝; 80¾.
32. Reduce to improper fractions 100⅞; 90¾; 60⅝; 80⅘.
33. Reduce to improper fractions 18⅘; 22⅝; 139⅘; 99¾.
34. Reduce to improper fractions 202₁₁²; 87½; 88⅞; 77¾.

35. A wheelman travels 16⅘ miles in an hour; how far does he go in one tenth of an hour?

MENTAL ARITHMETIC.

V. REDUCTION TO HIGHER TERMS.

1. How many sixths in an apple?

2. How many sixths in $\frac{1}{2}$ of an apple?

3. How many sixths in $\frac{1}{3}$ of an apple?

4. How many sixths in $\frac{2}{3}$?

SOLUTION.—In 1 there are 6 sixths; in $\frac{1}{3}$ there is $\frac{1}{3}$ of 6 sixths, which is 2 sixths; and in $\frac{2}{3}$ there are 2 times 2 sixths, or 4 sixths. Therefore, in $\frac{2}{3}$ there are $\frac{4}{6}$.

5. How many eighths in $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{3}{4}$? In $\frac{5}{8}$?

6. How many tenths in $\frac{1}{2}$? In $\frac{2}{5}$? In $\frac{3}{5}$? In $\frac{4}{5}$?

7. How many ninths in $\frac{1}{3}$? In $\frac{2}{3}$? In $\frac{1}{4}$? In $\frac{5}{6}$?

8. How many twelfths in $\frac{1}{2}$? In $\frac{2}{3}$? In $\frac{5}{6}$? In $\frac{3}{4}$?

9. How many sixteenths in $\frac{1}{4}$? In $\frac{3}{4}$? In $\frac{5}{8}$? In $\frac{7}{8}$?

10. Reduce $\frac{1}{2}$, $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{4}{5}$ to twentieths.

11. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{9}$ to eighteenths.

12. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{15}$, and $\frac{4}{5}$ to thirtieths.

13. Reduce $\frac{3}{8}$, $\frac{1}{4}$, $\frac{7}{10}$, and $\frac{3}{5}$ to fortieths.

14. Reduce $\frac{3}{4}$ to twelfths.

15. By what number must we multiply both numerator and denominator of $\frac{3}{4}$ to obtain $\frac{9}{12}$?

16. By what number must we multiply both numerator and denominator of $\frac{5}{8}$ to obtain $\frac{15}{24}$? To obtain $\frac{10}{16}$? To obtain $\frac{25}{40}$?

17. By what number must we multiply both numerator and denominator of $\frac{3}{4}$ to reduce it to 12ths? To 16ths? To 20ths?

NOTE.—To reduce a fraction to higher terms, *multiply both numerator and denominator by a number that will produce the given denominator.*

18. Reduce $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{7}{10}$ to 50ths.

19. Reduce $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$ to 36ths.

20. Reduce $\frac{4}{5}$, $\frac{7}{15}$, and $\frac{5}{9}$ to 45ths.

21. Reduce $\frac{5}{16}$, $\frac{7}{20}$, and $\frac{7}{8}$ to 80ths.

22. Reduce $\frac{2}{5}$ to 40ths. To 50ths. To 80ths.

23. Reduce $\frac{7}{8}$ to 32ds. To 48ths. To 64ths.

24. Reduce $\frac{5}{9}$ to 36ths. To 63ds. To 90ths.

25. Reduce $\frac{4}{11}$ to 55ths. To 88ths. To 121sts.

26. To what common denominator can $\frac{1}{2}$ and $\frac{1}{3}$ be reduced? What is the least common denominator to which they may both be reduced?

27. Reduce ⅔ and ¾ to their least common denominator.

SOLUTION.—The least common denominator of thirds and fourths is twelfths. ⅔ equal 𝒻₁₂, and ¾ equal 𝒻₁₂. Therefore, etc.

28. Reduce ⅓ and ½ to their least common denominator.
29. Reduce ¼ and ⅙ to their least common denominator.
30. Reduce ⅔ and ⅜ to their least common denominator.
31. Reduce ¾ and ⅚ to their least common denominator.
32. Reduce ⅔ and ⅝ to their least common denominator.
33. Reduce ⅗ and ⅚ to their least common denominator.
34. Reduce ⅖ and ₇/₁₀ to their least common denominator.
35. Reduce ⅚ and ₇/₁₂ to their least common denominator.

Reduce the following fractions to their least common denominators:

36. ½, ⅓, and ¼.	*41.* ½, ⅔, and ₇/₁₀.
37. ⅓, ¼, and ⅙.	*42.* ⅓, ⅚, and ₉/₁₂.
38. ½, ⅓, and ¼.	*43.* ⅘, ⅜, and ₉/₁₂.
39. ¼, ⅓, and ⅙.	*44.* ⅝, ⅞, and ⅚.
40. ⅔, ¾, and ⅝.	*45.* ⅔, ₅/₁₂, and ₇/₁₅.

VI. REDUCTION TO LOWER TERMS.

1. How many tenths in ½? How many fifths in ₄/₁₀? In ₆/₁₀? In ₈/₁₀?

2. How many fourteenths in ½? How many sevenths in ₄/₁₄? In ₆/₁₄? In ₈/₁₄? In ₁₂/₁₄?

3. How many thirds in ₈/₁₂?

SOLUTION.—Since ⅓ equals 𝒻₁₂, there are as many thirds in 𝒻₁₂ as 4 is contained times in 8, which are 2. Therefore, in 𝒻₁₂ there are ⅔.

4. How many halves in ⅔? ⁴⁄₄? ₆/₁₀? ⁸⁄₈? ¹⁰⁄₂?

5. How many thirds in ⁶⁄₉? ⁴⁄₄? ⁸⁄₉? ₉/₁₂? ¹²⁄₈?

6. How many fourths in ₄/₁₂? ₉/₁₂? ¹⁶⁄₈? ¹²⁄₈? ³⁰⁄₈?

7. How many fifths in ₄/₁₀? ₆/₁₅? ₈/₂₀? ¹⁵⁄₈? ¹²⁄₈?

8. How many sixths in ₂/₁₂? ¹⁸⁄₈? ²⁰⁄₂? ³⁶⁄₅? ³⁰⁄₈?

9. How many sevenths in ₄/₁₄? ₆/₂₁? ¹²⁄₈? ²⁸⁄₅? ³²⁄₇?

10. In reducing ⁴⁄₈ to ½, by what do we divide both numerator and denominator?

11. By what can we divide both numerator and denominator of ₆/₁₀? ¹⁸⁄₈? ⁶⁄₈?

NOTE.—To reduce a fraction to lower terms, divide both numerator and denominator by the same number.

12. How many eighths in $\frac{5}{16}$? $\frac{11}{2}$? $\frac{13}{8}$? $\frac{15}{8}$? $\frac{21}{4}$?

13. How many ninths in $\frac{5}{27}$? $\frac{13}{18}$? $\frac{25}{18}$? $\frac{41}{18}$? $\frac{9}{2}$?

14. How many tenths in $\frac{5}{40}$? $\frac{21}{5}$? $\frac{13}{5}$? $\frac{15}{5}$? $\frac{15}{8}$?

15. How many elevenths in $\frac{9}{33}$? $\frac{31}{4}$? $\frac{19}{8}$? $\frac{9\cdot3}{77}$? $\frac{55}{4}$?

16. How many twelfths in $\frac{19}{2}$? $\frac{11}{4}$? $\frac{35}{8}$? $\frac{41}{2}$? $\frac{99}{108}$?

NOTE.—A fraction is in its lowest terms when the numerator and the denominator contain no common factor.

17. Reduce $\frac{3}{6}$ and $\frac{5}{15}$ to their lowest terms.

18. Reduce $\frac{12}{18}$ and $\frac{21}{24}$ to their lowest terms.

19. Reduce $\frac{12}{18}$ and $\frac{12}{15}$ to their lowest terms.

20. Reduce $\frac{24}{36}$ and $\frac{16}{36}$ to their lowest terms.

21. Reduce $\frac{12}{18}$ and $\frac{21}{24}$ to their lowest terms.

22. Reduce $\frac{14}{28}$ and $\frac{21}{24}$ to their lowest terms.

23. Reduce $\frac{36}{48}$ and $\frac{21}{24}$ to their lowest terms.

24. Reduce $\frac{9}{12}$ and $\frac{77}{11}$ to their lowest terms.

25. Reduce $\frac{84}{96}$ and $\frac{16}{24}$ to their lowest terms.

26. Reduce $\frac{18}{48}$ and $\frac{84}{96}$ to their lowest terms.

27. Reduce $\frac{49}{56}$ and $\frac{81}{108}$ to their lowest terms.

28. Reduce $\frac{48}{56}$ and $\frac{48}{56}$ to their lowest terms.

29. Reduce $\frac{56}{64}$ and $\frac{42}{48}$ to their lowest terms.

30. Reduce $\frac{14}{21}$ and $\frac{63}{81}$ to their lowest terms.

VII. ADDITION OF FRACTIONS.

1. What is the sum of $\frac{2}{3}$ and $\frac{1}{4}$?

2. What is the sum of $\frac{1}{4}$ and $\frac{3}{4}$?

3. What is the sum of $\frac{3}{4}$ and $\frac{1}{5}$?

4. What is the sum of $\frac{4}{5}$ and $\frac{3}{8}$?

5. What is the sum of $\frac{7}{11}$ and $\frac{8}{11}$?

6. What is the sum of $\frac{4}{13}$, $\frac{6}{13}$, and $\frac{2}{13}$?

7. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$?

SOLUTION.—$\frac{1}{2}$ equals $\frac{3}{6}$, and $\frac{1}{3}$ equals $\frac{2}{6}$; $\frac{3}{6}$ plus $\frac{2}{6}$ are $\frac{5}{6}$. Therefore, etc.

8. Add $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{6}$; $\frac{1}{4}$ and $\frac{1}{5}$.

9. Add $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{5}$ and $\frac{1}{4}$.

10. Add $\frac{1}{6}$ and $\frac{1}{7}$; $\frac{1}{6}$ and $\frac{1}{5}$; $\frac{1}{4}$ and $\frac{1}{7}$.

11. Add $\frac{1}{8}$ and $\frac{1}{10}$; $\frac{1}{2}$ and $\frac{1}{7}$; $\frac{1}{7}$ and $\frac{1}{8}$.

NOTE.—Two fractions whose numerators are each *one* may be added by taking the sum of the denominators for the numerator, and their product for the denominator, of the result. Thus, $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.

What is the sum—

12. Of $\frac{1}{4}$ and $\frac{1}{6}$?	*18.* Of $\frac{1}{7}$ and $\frac{1}{11}$?
13. Of $\frac{1}{5}$ and $\frac{1}{4}$?	*19.* Of $\frac{1}{8}$ and $\frac{1}{12}$?
14. Of $\frac{1}{6}$ and $\frac{1}{10}$?	*20.* Of $\frac{1}{7}$ and $\frac{1}{8}$?
15. Of $\frac{1}{4}$ and $\frac{1}{12}$?	*21.* Of $\frac{1}{11}$ and $\frac{1}{12}$?
16. Of $\frac{1}{4}$ and $\frac{1}{9}$?	*22.* Of $\frac{1}{4}$ and $\frac{1}{15}$?
17. Of $\frac{1}{5}$ and $\frac{1}{10}$?	*23.* Of $\frac{1}{10}$ and $\frac{1}{20}$?

NOTE.—In like manner, $\dfrac{2}{3} + \dfrac{2}{7} = \dfrac{2 \times (3 + 7)}{3 \times 7} = \dfrac{20}{21}$.

What is the sum—

24. Of $\frac{3}{4}$ and $\frac{2}{6}$?	*28.* Of $\frac{4}{5}$ and $\frac{3}{4}$?
25. Of $\frac{3}{4}$ and $\frac{2}{7}$?	*29.* Of $\frac{9}{7}$ and $\frac{6}{11}$?
26. Of $\frac{3}{4}$ and $\frac{3}{7}$?	*30.* Of $\frac{5}{6}$ and $\frac{5}{8}$?
27. Of $\frac{5}{6}$ and $\frac{5}{6}$?	*31.* Of $\frac{7}{10}$ and $\frac{7}{11}$?

32. **What is the sum of $\frac{3}{4}$ and $\frac{5}{8}$?**

SOLUTION.—$\frac{3}{4}$ equal $\frac{10}{16}$, and $\frac{5}{8}$ equal $\frac{6}{16}$; $\frac{10}{16}$ plus $\frac{6}{16}$ are $\frac{16}{16}$, or $1\frac{6}{16}$. Therefore, etc.

What is the sum—

33. Of $\frac{3}{4}$ and $\frac{5}{7}$?	*43.* Of $\frac{7}{16}$ and $\frac{5}{24}$?
34. Of $\frac{3}{4}$ and $\frac{3}{7}$?	*44.* Of $\frac{7}{8}$ and $\frac{5}{6}$?
35. Of $\frac{7}{10}$ and $\frac{4}{15}$?	*45.* Of $\frac{3}{8}$ and $\frac{5}{7}$?
36. Of $\frac{5}{16}$ and $\frac{7}{8}$?	*46.* Of $\frac{7}{8}$ and $\frac{5}{8}$?
37. Of $\frac{5}{14}$ and $\frac{3}{21}$?	*47.* Of $\frac{3}{4}$ and $\frac{7}{11}$?
38. Of $\frac{3}{4}$ and $\frac{4}{5}$?	*48.* Of $\frac{3}{4}$ and $\frac{7}{16}$?
39. Of $\frac{3}{4}$ and $\frac{5}{8}$?	*49.* Of $\frac{5}{8}$ and $\frac{5}{27}$?
40. Of $\frac{3}{4}$ and $\frac{5}{8}$?	*50.* Of $\frac{8}{15}$ and $\frac{7}{25}$?
41. Of $\frac{5}{12}$ and $\frac{7}{8}$?	*51.* Of $\frac{7}{8}$ and $\frac{11}{12}$?
42. Of $\frac{5}{6}$ and $\frac{3}{8}$?	*52.* Of $\frac{4}{11}$ and $\frac{11}{4}$?

What is the sum—

53. Of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$?	*56.* Of $\frac{3}{7}$, $\frac{2}{3}$, and $\frac{5}{15}$?
54. Of $\frac{1}{3}$, $\frac{5}{6}$, and $\frac{4}{5}$?	*57.* Of $\frac{7}{8}$, $\frac{5}{11}$, and $\frac{7}{35}$?
55. Of $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{4}{15}$?	*58.* Of $\frac{5}{8}$, $\frac{3}{27}$, and $\frac{3}{4}$?

59. Harold gave $\frac{1}{3}$ of a quart of nuts to William, and $\frac{3}{4}$ of a quart to John; how many quarts of nuts did he give to both?

60. A book cost $\$\frac{3}{5}$, and a slate $\$\frac{3}{10}$; how much did both cost?

61. May read ⅔ of a book on Monday, and ⅛ of it on Tuesday; how much of the book did she read in the two days?

62. A spends ⅜ of his time in study, and ⅔ of it at work; what part of his time is occupied?

63. What is the sum of 2¼ and 3½?

SOLUTION.—¼ plus ½ equal ¾; 2 plus 3 are 5; hence, 2¼ plus 3½ equal 5¾.

What is the sum—

64. Of 2¼ and 5½?	*70.* Of 8¾ and 7⅞?
65. Of 3¼ and 2¾?	*71.* Of 4⅞ and 5⅜?
66. Of 4¼ and 5½?	*72.* Of 1⅝ and 7⅞?
67. Of 6⅔ and 3¼?	*73.* Of 6¾ and 8₁₀⁹?
68. Of 7½ and 5⅔?	*74.* Of 10½ and 12⅘?
69. Of 7¾ and 6⅝?	*75.* Of 20¾ and 30⅞?

76. A's hat cost $3½, and his coat $9¾; how much did both cost?

77. B bought 4¼ tons of coal at one time, and 6¾ tons at another time; how many tons did he buy?

78. E owns 5¾ acres of land, and B owns 4¼ acres; how many acres do they together own?

79. A farmer sowed 4½ acres in corn, 5¼ acres in wheat, and 6¼ acres in grass; how many acres did he sow?

80. A lady spent $4½ at one store, $7¼ at a second store, and $9¼ at a third store; how much did she spend at the three stores?

81. A boy walked 10½ miles on Monday, 12¾ miles on Tuesday, and 15¼ miles on Wednesday; how far did he walk in the three days?

VIII. SUBTRACTION OF FRACTIONS.

1. What is the difference between ⅔ and ½?

2. What is the difference between ¾ and ¼?

3. What is the difference between ⅜ and ⅔?

4. What is the difference between ⅝ and ⅔?

5. What is the difference between ⅞ and ⅜?

6. What is the difference between ⅝ and ⅓?

7. What is the difference between ₁₀⁷ and ₁₀⁹?

8. What is the difference between ½ and ¼?

SOLUTION.—½ equals ⅜, and ¼ equals ⅔; ⅜ minus ⅔ are ¼. Therefore, etc.

What is the difference—

9. Between $\frac{1}{2}$ and $\frac{1}{4}$?

10. Between $\frac{1}{3}$ and $\frac{1}{5}$?

11. Between $\frac{1}{4}$ and $\frac{1}{5}$?

12. Between $\frac{1}{4}$ and $\frac{1}{6}$?

13. Between $\frac{1}{3}$ and $\frac{1}{7}$?

14. Between $\frac{1}{5}$ and $\frac{1}{8}$?

15. Between $\frac{1}{6}$ and $\frac{1}{7}$?

16. Between $\frac{1}{4}$ and $\frac{1}{6}$?

NOTE.—Two fractions whose numerators are each 1 may be subtracted by taking the difference of the denominators for the numerator, and their product for the denominator, of the result. Thus, $\dfrac{1}{3} - \dfrac{1}{4} = \dfrac{4-3}{3 \times 4} = \dfrac{1}{12}$.

What is the difference—

17. Between $\frac{1}{3}$ and $\frac{1}{6}$?

18. Between $\frac{1}{4}$ and $\frac{1}{6}$?

19. Between $\frac{1}{5}$ and $\frac{1}{10}$?

20. Between $\frac{1}{4}$ and $\frac{1}{11}$?

21. Between $\frac{1}{4}$ and $\frac{1}{12}$?

22. Between $\frac{1}{4}$ and $\frac{1}{7}$?

23. Between $\frac{1}{2}$ and $\frac{1}{6}$?

24. Between $\frac{1}{5}$ and $\frac{1}{8}$?

NOTE.—In like manner, $\dfrac{2}{3} - \dfrac{2}{5} = \dfrac{2 \times (5-3)}{3 \times 5} = \dfrac{4}{15}$.

What is the difference—

25. Between $\frac{3}{4}$ and $\frac{3}{7}$?

26. Between $\frac{2}{3}$ and $\frac{2}{5}$?

27. Between $\frac{4}{7}$ and $\frac{4}{9}$?

28. Between $\frac{3}{5}$ and $\frac{3}{7}$?

29. Between $\frac{5}{6}$ and $\frac{5}{7}$?

30. Between $\frac{3}{8}$ and $\frac{3}{10}$?

31. Between $\frac{4}{7}$ and $\frac{4}{9}$?

32. Between $\frac{5}{8}$ and $\frac{5}{9}$?

33. **What is the difference between $\frac{3}{4}$ and $\frac{3}{8}$?**

SOLUTION.—$\frac{3}{4}$ equal $\frac{12}{16}$, and $\frac{3}{8}$ equal $\frac{6}{16}$; $\frac{12}{16}$ minus $\frac{6}{16}$ equal $\frac{6}{16}$. Therefore, etc.

What is the difference—

34. Between $\frac{3}{4}$ and $\frac{2}{3}$?

35. Between $\frac{3}{4}$ and $\frac{2}{3}$?

36. Between $\frac{3}{8}$ and $\frac{1}{2}$?

37. Between $\frac{3}{4}$ and $\frac{5}{6}$?

38. Between $\frac{5}{8}$ and $\frac{3}{7}$?

39. Between $\frac{5}{6}$ and $\frac{4}{5}$?

40. Between $\frac{2}{3}$ and $\frac{3}{8}$?

41. Between $\frac{5}{8}$ and $\frac{4}{7}$?

42. Between $\frac{3}{4}$ and $\frac{3}{7}$?

43. Between $\frac{7}{8}$ and $\frac{3}{4}$?

44. Between $\frac{9}{10}$ and $\frac{3}{4}$?

45. Between $1\frac{1}{2}$ and $\frac{3}{8}$?

46. Between $\frac{5}{8}$ and $\frac{3}{4}$?

47. Between $\frac{5}{8}$ and $\frac{5}{12}$?

48. Between $\frac{3}{4}$ and $\frac{7}{10}$?

49. Between $1\frac{1}{2}$ and $\frac{5}{8}$?

50. Between $\frac{5}{8}$ and $\frac{8}{11}$?

51. Between $\frac{5}{8}$ and $\frac{4}{5}$?

52. John gave his sister $\frac{9}{10}$ of a dollar, and his brother $\frac{3}{4}$ of a dollar; how much more did the sister receive than the brother?

53. Charles had $\frac{7}{10}$ of a dollar, and lost $\frac{2}{5}$ of a dollar; how much had he remaining?

54. A boy went to a store with $\frac{3}{4}$ of a dollar, and spent $\frac{2}{3}$ of a dollar; how much had he remaining?

55. William had a melon, and gave $\frac{1}{2}$ of it to his companions, and $\frac{1}{3}$ of it to his brother; how much had he remaining?

56. $\frac{1}{3}$ of a field is planted in corn, $\frac{1}{4}$ of it in wheat, and the remainder in oats; how much of the field is in oats?

57. $\frac{2}{3}$ of an army were killed in battle, and $\frac{1}{10}$ of it deserted; what part of the army remained?

58. What is the difference between $5\frac{1}{2}$ and $3\frac{1}{4}$?

SOLUTION.—$\frac{1}{2}$ minus $\frac{1}{4}$ equal $\frac{1}{4}$; 5 minus 3 equal 2; hence, $5\frac{1}{2}$ minus $3\frac{1}{4}$ equal $2\frac{1}{4}$.

What is the difference—

59. Between $3\frac{2}{3}$ and $2\frac{1}{2}$?

60. Between $5\frac{3}{5}$ and $4\frac{1}{2}$?

61. Between $7\frac{3}{5}$ and $5\frac{1}{4}$?

62. Between $10\frac{5}{6}$ and $8\frac{3}{4}$?

63. Between $12\frac{7}{8}$ and $5\frac{1}{4}$?

64. Between $14\frac{3}{5}$ and $5\frac{2}{3}$?

65. Between $20\frac{3}{4}$ and $12\frac{3}{4}$?

66. Between $18\frac{5}{7}$ and $10\frac{3}{4}$?

67. Between $16\frac{1}{4}$ and $7\frac{3}{10}$?

68. Between $22\frac{3}{8}$ and $9\frac{1}{3}$?

69. What is the difference between $6\frac{3}{8}$ and $2\frac{3}{4}$?

SUGGESTION.—$6\frac{3}{8}$ equal $5\frac{11}{8}$; $5\frac{11}{8}$ minus $2\frac{3}{4}$ equal $3\frac{11}{8}$.

70. Between $9\frac{3}{8}$ and $6\frac{5}{7}$?

71. Between $11\frac{1}{4}$ and $8\frac{3}{4}$?

72. Between $6\frac{3}{10}$ and $4\frac{4}{5}$?

73. Between $8\frac{1}{4}$ and $3\frac{4}{5}$?

74. Between $9\frac{3}{11}$ and $2\frac{4}{5}$?

75. Between $10\frac{3}{4}$ and $3\frac{9}{10}$?

76. $3\frac{7}{8}$ gallons of water were taken from a vessel containing $12\frac{5}{8}$ gallons; how much water remained?

77. B is $37\frac{1}{2}$ years old, and his son $13\frac{3}{4}$ years; what is the difference in their ages?

78. Mr. Smith, having $31\frac{4}{5}$ acres of land, sold $16\frac{3}{4}$ acres; how many acres had he remaining?

79. John bought a horse for $\$120\frac{4}{5}$, and sold him for $\$130\frac{1}{2}$; how much did he gain?

80. A has $\$24\frac{1}{2}$: after paying $\$3\frac{1}{4}$ for a hat, and $\$6\frac{1}{2}$ for a pair of shoes, how much will he have remaining?

81. From a field containing $30\frac{4}{5}$ acres, A sold $10\frac{5}{8}$ acres; how many acres remained?

What is the value—

82. Of $\frac{1}{2} - \frac{1}{4} + \frac{1}{2}$?	*87.* Of $2\frac{1}{2} + 3\frac{1}{2} - 4\frac{1}{4}$?
83. Of $\frac{1}{3} + \frac{1}{4} - \frac{1}{3}$?	*88.* Of $6\frac{1}{2} - 2\frac{1}{2} + 5\frac{3}{4}$?
84. Of $\frac{2}{3} - \frac{3}{8} + \frac{1}{2}$?	*89.* Of $7\frac{1}{2} - 5\frac{3}{8} + 5\frac{7}{10}$?
85. Of $\frac{1}{2} + \frac{5}{8} - \frac{7}{10}$?	*90.* Of $4\frac{1}{4} + 5\frac{1}{2} - 6\frac{1}{2}$?
86. Of $\frac{2}{3} + \frac{1}{2} - \frac{3}{4}$?	*91.* Of $6\frac{7}{8} - 1\frac{3}{4} + 4\frac{1}{16}$?

IX. MULTIPLICATION OF FRACTIONS.

1. What are $\frac{1}{4} + \frac{1}{4}$? What are 2 times $\frac{1}{4}$?

2. What are $\frac{2}{7} + \frac{2}{7} + \frac{2}{7}$? What are 3 times $\frac{2}{7}$?

3. What are $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$? What are 4 times $\frac{2}{5}$?

4. What are 5 times $\frac{2}{3}$?

SOLUTION.—5 times $\frac{2}{3}$ are $\frac{10}{3}$, or $3\frac{1}{3}$. Therefore, etc.

5. What are 3 times $\frac{3}{8}$? 4 times $\frac{2}{3}$? 5 times $\frac{3}{4}$?

6. What are 3 times $\frac{4}{7}$? 4 times $\frac{5}{6}$? 5 times $\frac{4}{9}$?

7. What are 6 times $\frac{2}{3}$? 8 times $\frac{4}{5}$? 10 times $\frac{3}{8}$?

8. What are 5 times $\frac{4}{9}$? 7 times $\frac{3}{8}$? 9 times $\frac{3}{5}$?

9. What are 6 times $\frac{5}{9}$? 9 times $\frac{4}{5}$? 10 times $\frac{5}{9}$?

10. What are 8 times $\frac{4}{7}$? 11 times $\frac{3}{8}$? 12 times $\frac{7}{11}$?

11. What are 4 times $\frac{3}{16}$?

Since 4 times $\frac{3}{16}$ equal $\frac{12}{16}$, or $\frac{3}{4}$, it is seen that a fraction may be multiplied either by multiplying the numerator or dividing the denominator.   Hence, we have the following brief

SOLUTION.—4 times $\frac{3}{16}$ equal $\frac{3}{4}$.

12. What are 5 times $\frac{3}{10}$? 6 times $\frac{5}{12}$? 7 times $\frac{5}{21}$?

13. What are 6 times $\frac{5}{18}$? 8 times $\frac{7}{16}$? 9 times $\frac{5}{18}$?

14. What are 4 times $1\frac{1}{2}$? 6 times $1\frac{3}{4}$? 12 times $\frac{5}{12}$?

15. What are 5 times $\frac{7}{60}$? 10 times $\frac{19}{100}$? 12 times $\frac{7}{60}$?

16. What are 7 times $\frac{5}{42}$? 9 times $1\frac{5}{18}$? 10 times $1\frac{7}{15}$?

17. What are 13 times $\frac{5}{39}$? 15 times $\frac{7}{45}$? 12 times $\frac{40}{144}$?

18. What are 8 times $3\frac{2}{3}$?

SOLUTION.—8 times $\frac{2}{3}$ are $\frac{16}{3}$, or $5\frac{1}{3}$; 8 times 3 are 24; hence, 8 times $3\frac{2}{3}$ are 24 *plus* $5\frac{1}{3}$, or $29\frac{1}{3}$.

19. What are 6 times $4\frac{1}{2}$? 5 times $2\frac{3}{4}$? 7 times $3\frac{1}{2}$?

20. What are 7 times $5\frac{1}{2}$? 9 times $4\frac{2}{3}$? 10 times $3\frac{3}{4}$?

21. What are 8 times 6⅞?   10 times 3₁⁸₀?   12 times 9½?

22. What are 7 times 4⅝?   8 times 6⅞?   9 times 4⅝?

What is the cost—

23. Of 8 pounds of sugar, at 5½ cents a pound?

24. Of 5 cords of wood, at $5½ a cord?

25. Of 6 tons of coal, at $5¼ a ton?

26. Of 9 gallons of oil, at $1⅘ a gallon?

27. Of 12 yards of ribbon, at 6½ cents a yard?

28. Of 10 barrels of flour, at $3¼ a barrel?

29. Of 9 yards of cloth, at $2¼ a yard?

30. Of 4 tons of hay, at $16¾ a ton?

31. Of 5 acres of land, at $80⅘ an acre?

32. Of 6 sheep, at $8⅘ apiece?

33. What are ⅔ of 8?

SOLUTION.—⅓ of 8 is ⅘, and ⅔ of 8 are 2 times ⅘, which are ¹⁶⁄₃, or 5⅓. Therefore, etc.

34. What is ½ of 7?   ⅓ of 13?   ¼ of 15?

35. What is ⅕ of 50?   ⅐ of 70?   ₁₀ of 66?

36. What are ⅔ of 13?   ¾ of 18?   ⅜ of 8?

37. What are ⅞ of 9?   ⅚ of 14?   ⅘ of 12?

38. What are ⅔ of 17?   ⅜ of 8?   ¹⁰⁄₁₃ of 13?

39. What are ⅘ of 40?   ⅚ of 21?   ⁵⁄₉ of 36?

40. Charles has $14, and Henry has ¾ as much; how much money has Henry?

41. A merchant bought flour at $4 a barrel, and sold it at ⅚ of the cost; how much did he receive for it?

42. C bought 60 bushels of apples, and sold ⅘ of them; how many bushels did he sell? How many bushels remained?

43. A boy had $8, and spent ₁⁷₀ of it; how much did he spend?

44. A farmer sold ⅝ of a farm containing 108 acres; how many acres had he remaining?

45. A kite was 120 feet from the ground, and fell ⅘ of this distance; how far was it from the ground then?

46. What are ⅔ of ⅘?

SOLUTION.—⅓ of ⅕ is ₁⅕; ⅓ of ⅘ is 4 times ₁⅕, or ₁⅘; and ⅔ of ⅘ are 2 times ₁⅘, or ₁⅘. Therefore, etc.

47. What is ½ of ⅓?   ⅓ of ⅖?   ¼ of ⅘?

48. What are ¼ of ⁷⁄₈? ⅝ of ⅔? ⁴⁄₇ of ⅚?
49. What are ₁⁵ᵣ of ⅔? ⅞ of ⅝? ⅘ of ⁴⁄₇?
50. What are ⅝ of ⁴⁄₇?

SUGGESTION.—In practice we say, ⅝ of ⁴⁄₇ equal ²⁰⁄₅₆.

51. What are ⁴⁄₇ of ⅝? ⁷⁄₄ of ⅔? ⅝ of ⅝?
52. What are ⅝ of ⁴⁄₇? ⁹⁄₁₀ of ⁷⁄₁₁? ⁴⁄₇ of ₁³ᵣ?
53. What are ⁹⁄₁₀ of 1½? ₁¹ᵣ of ⁷⁄₇? ⁴⁄₇ of ⅝?
54. What are ⅘ of 3½?

SUGGESTION.—3½ equal ⁷⁄₂; hence, ⅘ of 3½ are ⅘ of ⁷⁄₂.

55. What are ⅔ of 3½? ¾ of 4⅕? ⅔ of 6¼?
56. What are ⅘ of 2½? ⅝ of 3¾? ⁷⁄₈ of 5½?
57. What are ⅜ of 12½? ⁷⁄₉ of 4⅔? ⁴⁄₇ of 5¼?
58. What will ⅔ of a ton of coal cost, at $5¼ a ton?
59. A horse eats 5½ quarts of oats in a day; how much will he eat in 9 days?
60. A man earns $2½ a day; how much will he earn in 5½ days?
61. A can travel 5¾ miles in an hour; how far can he travel in ⅝ of an hour?
62. How much will 2½ dozen of eggs cost, at 15¼ cents a dozen?
63. What is the cost of 6½ pounds of sugar, at 5¼ cents a pound?

X. DIVISION OF FRACTIONS.

1. What are 4 times ⅜? What are ⅝ divided by 4?
2. What are 5 times ⅘? What are ²⁰⁄₆ divided by 5?
3. What are 4 times ⅔? What are ⅝ divided by 4?

NOTE.—From this it will be seen that dividing the numerator of a fraction divides the value of the fraction.

4. Divide ¹²⁄₉ by 4.

SOLUTION.—¹²⁄₉ divided by 4 equal ⅜.

5. Divide ⅜ by 3. ⅜ by 2. ⁴⁄₉ by 3.
6. Divide ¹²⁄₉ by 4. ¹⁵⁄₄ by 5. ²⁰⁄₁₁ by 5.
7. Divide ³⁶⁄₈ by 3. ³⁵⁄₉ by 7. ³²⁄₇ by 8.
8. Divide ⁴⁰⁄₉ by 8. ⁴⁴⁄₅ by 9. ⁴⁸⁄₅ by 12.
9. Divide ⁶⁴⁄₉ by 7. ⁸¹⁄₇ by 9. ⁷²⁄₇ by 8.
10. Divide ²²⁄₉ by 11. ¹²⁰⁄₇ by 12. ¹⁴⁴⁄₇ by 12.

11. What are 5 times $\frac{1}{15}$? What are $\frac{3}{4}$ divided by 5?
12. What are 6 times $\frac{1}{20}$? What are $\frac{4}{5}$ divided by 6?
13. What are 7 times $\frac{5}{42}$? What are $\frac{5}{6}$ divided by 7?

NOTE.—From this it will be seen that multiplying the denominator of a fraction divides the value of the fraction.

14. Divide $\frac{3}{4}$ by 5.

SOLUTION.—$\frac{3}{4}$ divided by 5 equal $\frac{3}{20}$.

15. Divide $\frac{3}{8}$ by 7.　$\frac{3}{4}$ by 5.　$\frac{4}{5}$ by 3.
16. Divide $\frac{2}{3}$ by 4.　$\frac{3}{5}$ by 5.　$\frac{5}{6}$ by 6.
17. Divide $\frac{5}{6}$ by 3.　$\frac{6}{7}$ by 7.　$\frac{9}{11}$ by 4.
18. Divide $\frac{6}{7}$ by 7.　$\frac{7}{8}$ by 9.　$\frac{9}{7}$ by 11.
19. Divide $\frac{8}{10}$ by 5.　$\frac{4}{15}$ by 3.　$\frac{3}{11}$ by 5.
20. Divide $\frac{8}{11}$ by 6.　$\frac{9}{13}$ by 7.　$\frac{4}{13}$ by 5.
21. Divide $2\frac{1}{2}$ by 5.

SOLUTION.—$2\frac{1}{2}$ equal $\frac{15}{6}$; $2\frac{1}{2}$ divided by 5 equal $\frac{15}{6}$ divided by 5, or $\frac{1}{2}$.

22. Divide $6\frac{1}{4}$ by 5.　$7\frac{1}{3}$ by 11.　$8\frac{3}{4}$ by 7.
23. Divide $2\frac{2}{5}$ by 3.　$3\frac{3}{4}$ by 5.　$5\frac{3}{4}$ by 7.
24. Divide $2\frac{2}{5}$ by 7.　$3\frac{1}{4}$ by 6.　$3\frac{1}{2}$ by 5.
25. Divide $8\frac{3}{4}$ by 6.　$5\frac{5}{6}$ by 3.　$8\frac{3}{4}$ by 6.

26. A man divided $\$\frac{15}{4}$ equally among 5 boys; what part of a dollar did each boy receive?

27. A divided $\$1\frac{3}{4}$ equally among 5 beggars; what part of a dollar did each beggar receive?

28. If a dozen eggs cost $\frac{3}{10}$ of a dollar, what will 1 egg cost?

29. A laborer earns $\$13\frac{1}{2}$ in 6 days; required his daily wages.

30. If 5 barrels of flour cost $\$18\frac{3}{4}$, what will 1 barrel cost?

31. B divides $6\frac{3}{4}$ acres of land into 5 equal lots; how much land in each lot?

32. If 6 sheep cost $\$37\frac{1}{2}$, what will 1 sheep cost?

33. If 6 tons of coal cost $\$31\frac{1}{2}$, what will 1 ton cost?

34. If 11 barrels of apples cost $\$41\frac{1}{4}$, what will 1 barrel cost?

35. Divide 3 by $\frac{2}{5}$.

SOLUTION.—3 divided by 1 is 3, and 3 divided by $\frac{1}{5}$ is 5 times 3, or 15; 3 divided by $\frac{2}{5}$ is $\frac{1}{2}$ of 15, or $\frac{15}{2}$, or $7\frac{1}{2}$. Therefore, etc.

36. Divide 4 by $\frac{3}{5}$.　5 by $\frac{3}{4}$.　6 by $\frac{5}{7}$.
37. Divide 7 by $\frac{3}{4}$.　8 by $\frac{4}{7}$.　9 by $\frac{5}{8}$.

38. Divide 10 by ⅘. 11 by ⅚. 12 by ⅞.

39. Divide 8 by ⅝. 9 by ⅔. 10 by ¹⁄₁₁.

40. Divide 6 by ¾. 12 by ¾. 8 by ⅝.

41. If 2½ pounds of sugar cost 14 cents, what will 1 pound cost?

42. If 3½ barrels of apples cost $12, what will 1 barrel cost?

43. If 8⅔ tons of coal cost $35, what will 1 ton cost?

44. How many bushels of wheat, at $¾ per bushel, can be bought for $16?

45. A boy shared 21 apples equally with his companions, giving to each 3½ apples; how many companions had he?

46. Mary gave 10 pounds of butter, at $³⁄₁₆ a pound, for ribbon, at $⅔ a yard; how many yards of ribbon did she receive?

47. A farmer exchanged 9 sheep, at $6¾ a head, for calves, at $3¾ each; how many calves did he get? | $\llcorner$ ¦⁻

48. Divide ⅔ by ⅘.

SOLUTION 1.—⅔ equal ¹⁰⁄₁₅; ⅘ equal ¹²⁄₁₅; ¹²⁄₁₅ are contained in ¹⁰⁄₁₅ as often as 15 is contained in 14, or ¹⁴⁄₁₅ times. Therefore, etc.

SOLUTION 2.—⅔ divided by 1 are ⅔, and ⅔ divided by ⅘ are 7 times ⅔, which are ¹⁴⁄₅; ⅔ divided by ⅘ are ⅕ of ¹⁴⁄₅, or ¹⁴⁄₁₅. Therefore, etc.

49. Divide ½ by ⅘. ⅖ by ⅘. ⅚ by ¾.

50. Divide ⅔ by ¼. ⅖ by ⅝. ⅝ by ⅝.

51. Divide ⅝ by ½. ¾ by ⅘. ⅖ by ⅝.

52. Divide ⅔ by ⅝. ⅞ by ¾. ⅝ by ⁵⁄₁₂.

NOTE.—In practice we invert the divisor, and proceed as in multiplication.

53. Divide ⅝ by ⅘. ⁷⁄₁₀ by ⅘. ¹⁄₁₁ by ⅘.

54. Divide ⅝ by ⅝. ⅔ by ⅜. ¼ by ⅘.

55. Divide ⁷⁄₁₂ by ⅝. ⅞ by ⅝. ¹⁄₁₁ by ⅘.

56. Divide ¹⁄₁₈ by ⅘. ⅝ by ¹⁄₁₁. ⁷⁄₁₂ by ³⁄₁₀.

57. Divide 2½ by 4⅔.

SUGGESTION.—2¼ equal ⅘; 4⅔ equal ¹⁴⁄₃; ⅘ divided by ¹⁴⁄₃ equal ⅔ times ³⁄₁₄, which are ⁹⁄₁₈, or ½.

58. Divide 3½ by ⅝. 2¼ by ⅝. 1⅔ by ⁷⁄₁₀.

59. Divide 1⅞ by ¾. 8½ by 2½. 4¼ by 5⅔.

60. A laborer received $4½ for 1½ days' work; required his daily wages.

61. If 5⅔ yards of cloth make one suit of clothes, how many suits can be made from 21⅔ yards?

62. At 12½ cents a quart, how many quarts of chestnuts can be bought for 1 dollar?

63. At ⅝ of a dollar a bushel, how many bushels of wheat can be bought for $2½?

64. If a man can do a piece of work in 12⅔ days, how many men will be required to do the same work in 2½ days?

65. A farmer sold 7½ pounds of butter, at ₃/₁₀ of a dollar a pound, and took in exchange muslin, at ⅓ of a dollar a yard; how many yards did he receive?

66. How many pounds of sugar, at ₁/₁₂ of a dollar a pound, can be bought for 6¼ bushels of potatoes, at ⅗ of a dollar a bushel?

XI. RELATION OF NUMBERS.

I. Integer to an Integer.

1. What part of 4 is 3? What part of 3 is 4?

SOLUTION 1.—1 is ¼ of 4, and 3 is 3 times ¼ of 4, or ¾ of 4. Therefore, etc.
SOLUTION 2.—1 is ⅓ of 3, and 4 is 4 times ⅓ of 3, or ⁴⁄₃ of 3, = 1⅓ times 3. Therefore, etc.

What part—

2. Of 12 is 3?	*9.* Of 75 is 35?
3. Of 16 is 4?	*10.* Of 72 is 81?
4. Of 18 is 5?	*11.* Of 60 is 33?
5. Of 24 is 7?	*12.* Of 120 is 150?
6. Of 9 is 27?	*13.* Of 200 is 160?
7. Of 10 is 45?	*14.* Of 96 is 72?
8. Of 6 is 27?	*15.* Of 80 is 32?

16. If 8 oranges cost 15 cents, what will 24 oranges cost?

SOLUTION.—If 8 oranges cost 15 cents, 24 oranges, which are 3 times 8 oranges, will cost 3 times 15 cents, or 45 cents. Therefore, etc.

17. If 12 pencils cost 10 cents, what will 60 pencils cost?

18. If 10 sheep cost $48, what will 25 sheep cost at the same rate?

19. If 17 hats cost $3, what will 51 hats cost?

20. What will 9 caps cost, if 36 caps cost $20?

21. What will 3 coats cost, if 21 coats cost $287?

22. What will 5 cows cost, if 10 cows cost $324?

23. If ⅓ of 12 bushels of grain cost $5, what will 24 bushels cost?

24. If 10 men build 14 rods of wall in a certain time, how many rods will 25 men build in the same time?

25. How far can a boy travel on his bicycle in 42 days, if he travels 70 miles in 6 days?

26. If 8 men earn $125 in a certain time, how much will 12 men earn in the same time?

27. If 10 men can do a piece of work in $4\frac{1}{2}$ days, in what time can 5 men do it?

28. If $\frac{2}{3}$ of 12 turkeys cost $15, what will $\frac{3}{4}$ of 32 turkeys cost?

29. If $\frac{3}{4}$ of 8 sheep cost $32, what will $\frac{2}{3}$ of 18 sheep cost?

II. Fraction to an Integer.

1. What part of 7 are $\frac{2}{3}$?

SOLUTION 1.—1 is $\frac{1}{7}$ of 7, and $\frac{1}{3}$ is $\frac{1}{3}$ of $\frac{1}{7}$, which is $\frac{1}{21}$ of 7, and $\frac{2}{3}$ are 2 times $\frac{1}{21}$, which are $\frac{2}{21}$ of 7. Therefore, etc.

SOLUTION 2.—7 equals $\frac{21}{3}$; hence, $\frac{2}{3}$ are $\frac{2}{21}$ of 7.

What part—

2. Of 7 are $\frac{2}{5}$?	*8.* Of 16 are $\frac{4}{7}$?
3. Of 9 are $\frac{3}{4}$?	*9.* Of 12 are $\frac{4}{5}$?
4. Of 11 are $\frac{3}{8}$?	*10.* Of 14 are $\frac{1}{5}$?
5. Of 4 are $\frac{7}{10}$?	*11.* Of 18 are $\frac{2}{3}$?
6. Of 5 are $\frac{9}{11}$?	*12.* Of 15 are $\frac{4}{9}$?
7. Of 6 are $\frac{9}{10}$?	*13.* Of 10 are $\frac{2}{3}$?

14. A can travel 18 miles in 5 hours; how far can he travel in $1\frac{3}{4}$ hours?

SOLUTION.—If A travels 18 miles in 5 hours, in $1\frac{3}{5}$, or $\frac{8}{5}$, hours, which are $\frac{1}{3}$ of 5 hours, he can travel $\frac{1}{3}$ of 18 miles, or 6 miles. Therefore, etc.

15. B saves money enough in 12 years to pay for his home; what part of it does he save in $1\frac{4}{5}$ years?

16. If 10 bushels of corn cost $5.13, what will $3\frac{1}{3}$ bushels cost?

17. If 16 yards of cloth cost $30, what will $3\frac{1}{5}$ yards cost?

18. A book was bought for $8, and sold at a gain of 2\frac{2}{3}$; what part of the cost equals the gain?

19. A bought goods for $12, but, finding them slightly damaged, sold them for 1\frac{1}{2}$ less than cost; what part of the cost did he lose?

20. A hat was bought for $3, and sold for 3\frac{3}{4}$; what part of the cost was gained?

21. If 15 acres of land cost $1206, what will 2½ acres cost?

22. If 100 feet of lumber cost $3.12, what will 33¼ feet cost?

23. If 52 pounds of sugar cost $2.75, what will 10⅔ pounds cost?

24. What will 12½ pounds of butter cost, if 50 pounds cost $12.04?

25. What will 16⅔ yards of muslin cost, if 50 yards cost $2.10?

III. Integer to a Fraction.

1. What part of ⅔ is 6?

SOLUTION 1.—⅓ is ½ of ⅔, and ⅔, or 1, are 3 times ⅓, or ½ of ⅔. Since 1 is ½ of ⅔, 6 is 6 times ½ of ⅔, or ¹⁸⁄₂ of ⅔, = 9 times ⅔. Therefore, etc.

SOLUTION 2.—6 equals ¹⁸⁄₃, and ¹⁸⁄₃ are 9 times ⅔. Therefore, 6 is 9 times ⅔.

What part—

2. Of ⅔ is 8?	*7.* Of ¹⁰⁄₁₁ is 25?
3. Of ¾ is 9?	*8.* Of ⅝ is 16?
4. Of ⅚ is 10?	*9.* Of ¾ is 20?
5. Of ⁷⁄₈ is 12?	*10.* Of ⁹⁄₁₀ is 18?
6. Of ⅜ is 14?	*11.* Of ⅝ is 24?

12. A had 6 acres of land, and B had ¾ as much plus ¼ of an acre; what part of B's land equals A's?

13. John had 10 quarts of chestnuts, and Harold had ⅚ as many plus ⅔ of a quart; what part of Harold's equals John's?

14. William is 12 years old, and Harry is ⅚ as old minus ⅔ of a year; what part of Harry's age equals William's?

15. George can do a piece of work in 8 days, and Charles in 3¼ days; how much faster does Charles work than George?

16. A farmer sold a sheep which cost him $5½ for $7½; what part of the cost equals the gain?

17. M paid $8½ for a lawn mower, and sold it for $5½; what part of the cost equals the loss?

18. What part of ⅔ of 5 are ¾ of 8?

19. What part of ⅚ of 9 are ⅔ of 6?

20. What part of ¾ of 5 are ⅝ of 12?

21. What part of ⅔ of 7 are ⁴⁄₇ of 14?

22. What part of ⅝ of 12 are ⅘ of 18?

23. What part of ⁷⁄₁₀ of 15 are ⅔ of 15?

24. What part of ⅔ of 8 are ¾ of 16?

IV. Fraction to a Fraction.

1. What part of ⅘ are ⅔?

SOLUTION 1.—⅕ is ½ of ⅖, and ⅖, or 1, are 7 times ½, or ⅞, of ⅘; ⅓ is ⅓ of ⅔, or ₁⁵₅, of ⅘, and ⅔ are 2 times ₁⁵₅, or ½⅔, of ⅘. Therefore, ⅔ are ½⅔ of ⅘.

SOLUTION 2.—⅘ equal ½⅘, and ⅔ equal ½⅔; then ½⅔ are ½⅔ ot ½⅘. Therefore, ⅔ are ½⅔ of ⅘.

What part—

2. Of ⅓ are ¼?
3. Of ⅖ are ⅝?
4. Of ⅘ are ⅝?
5. Of ⅖ are ⅐?
6. Of ⅔ are ⅘?
7. Of ⅝ are ⅘?
8. Of ⅐ are ⅝?
9. Of ⅘ are ⁰⁄₅?

10. Of ½ is ⅓?
11. Of ⅓ is ⅓?
12. Of ¼ is ⅐?
13. Of ⅞ are ¼?
14. Of ¾ are ⅝?
15. Of ⅘ are ₁³₁?
16. Of ⅓ are ⅝?
17. Of ⅜ is ⅓?

18. A bought a hat for $2½, and sold it for $3¼; what part of the cost equals the gain?

19. If an article is bought for $5¼, and sold for $6¾, what part of the cost equals the gain?

20. If lead pencils are bought at the rate of 3 for a cent, and sold at the rate of 2 for a cent, what part of the cost equals the gain?

21. A boy had ⅘ of a dollar, and spent ¼ of a dollar; what part of his money remained?

22. A book which cost $2½ was sold for $1¾; what part of the cost equals the loss?

23. What part of ⅔ of ¾ are ¾ of ⅝?

24. What part of ¼ of ⅖ is ⅓ of ⅔?

25. What part of ⅔ of ¾ is ⅓ of ⅓?

26. What part of ⅜ of ¾ are ⅔ of ⅝?

27. What part of ⅓ of ⅔ are ⅝ of ⅔?

28. What part of ⅔ of ⅝ are ¼ of ⅝?

XII. ARITHMETICAL ANALYSIS.

I. Integer compared with an Integer.

1. If 7 oranges cost 17½ cents, what will 6 oranges cost?

SOLUTION.—If 7 oranges cost 17½, or ³⁵⁄₂, cents, 1 orange will cost ⅐ of ³⁵⁄₂ cents, which is ⅝ cents; and 6 oranges will cost 6 times ⅝ cents, or 15 cents. Therefore, etc.

2. If 5 tons of coal cost $27½, what will 12 tons cost?

3. If 8 quarts of milk cost 44 cents, what will 3 quarts cost?

4. How much must I pay for 16 yards of muslin, if 9 yards cost 49¼ cents?

5. William earns $12½ in 5 days; how much can he earn in 22 days?

6. If 5 tons of hay cost $52½, what will 8 tons cost at the same rate?

7. If 9 turkeys cost $22½, what will 11 turkeys cost at the same rate?

8. What must I pay for 20 pounds of butter, if 6 pounds cost 19¼ dimes?

9. If 12 books cost $28⅘, what will be the cost of 15 books?

10. If boarding for 9 weeks costs $40½, how much will boarding for 12 weeks cost?

11. If 5 gallons of oil cost 26¼ cents, what will 14 gallons cost?

12. If 8 men can do a piece of work in 4½ days, how long will it take 3 men to do the same work?

II. Fraction compared with an Integer.

1. If ⅞ of a ton of hay cost $14, what will a ton cost?

SOLUTION.—If ⅞ of a ton cost $14, ⅛ of a ton will cost ⅐ of $14, which is $2; and ⅛, or a ton, will cost 8 times $2, which are $16. Therefore, etc.

2. If ⅘ of a ton of coal cost $4, what will a ton cost?

3. If ⅝ of a pound of butter cost 25 cents, what will a pound cost?

4. If ¾ of the price of a sheep are $9, what will 5 sheep cost?

5. What must I pay for 10 yards of cloth, if ⅞ of a yard cost $1.40?

6. What will 12 barrels of flour cost, if ¾ of a barrel cost $6?

7. What will 15 bushels of wheat cost, if 3¼ bushels cost $2.70?

8. If ₆/₁₀ of a barrel of sugar cost $8.40, what will 5 barrels cost?

9. If ⅝ of a cord of wood cost $3.50, what will 20 cords cost?

10. What must I pay for a farm of 100 acres, if 2½ acres cost $125?

11. If ⅘ of the cost of a horse are $120, what will 12 horses cost at the same rate?

12. If 12½ bushels of potatoes cost $10, what will 8 bushels cost?

III. Integer compared with a Fraction.

1. If 10 yards of cloth cost $30, what will ¾ of a yard cost?

SOLUTION.—If 10 yards of cloth cost $30, 1 yard will cost $\frac{1}{10}$ of $30, which is $3; ¼ of a yard will cost ¼ of $3, which is $1, and ¾ of a yard will cost 2 times $1, which are $2.

2. If 5 pounds of butter cost $1.50, what will 2¼ pounds cost?

3. If George earns $60 in 10 days, how much does he earn in 4½ days?

4. If 8 acres of land cost $400, what will 2⅔ acres cost?

5. When 5 pounds of tea cost $4, what do 3½ pounds cost?

6. If 10 quarts of ice cream cost $2.50, what will 4½ quarts cost?

7. When 12 pounds of beef cost $1.80, what do 6⅝ pounds cost at the same rate?

8. What will 2¾ acres of land cost, if 8 acres cost $480?

9. If 20 quires of paper cost $4.20, what will 5⅔ quires cost?

10. What must I pay for 5⅝ cords of wood, at the rate of 10 cords for $60?

11. What must I pay for ¾ of 5 yards of carpet, if 6 yards cost $5.40?

IV. Fraction compared with a Fraction.

1. If ⅗ of a ton of coal cost $3.60, what will ¾ of a ton cost?

SOLUTION.—If ⅗ of a ton of coal cost $3.60, ⅕ of a ton will cost ⅓ of $3.60, which is $1.20; and ⅖, or a ton, will cost 5 times $1.20, which are $6.

If 1 ton costs $6, ¼ of a ton will cost ¼ of $6, which is $1.50; and ¾ of a ton will cost 3 times $1.50, which are $4.50.

2. If ⅓ of a ton of coal costs $4, what will ⅚ of a ton cost?,

3. If 2½ barrels of apples cost $10, what will 3⅛ barrels cost?

4. If 4½ yards of cloth cost $18, what will 7¼ yards cost?

5. What must I pay for 6⅛ pounds of meat, at the rate of 2½ pounds for 37½ cents?

6. If ⅚ of A's age are 30 years, what are ⅘ of his age?

7. If 2⅔ tons of coal cost $13¼, what will 3¾ tons cost?

8. How many sheep, at $4½ apiece, can be bought for 8 tons of hay, at $9 a ton?

9. If A earns $12½ in 4 days, how much can he earn in 3½ days?

10. If 3½ dozen eggs cost 84 cents, what will 5¼ dozen cost?

11. What will ¾ of a yard of silk cost, if ⅔ of a yard cost $⅝?

XIII. MISCELLANEOUS PROBLEMS.

1. What is ⅓ of 24?

2. 24 is ⅓ of what number?

3. What part of 24 is 8?

4. 50 is ⅓ of what number?

5. What are ⅔ of 100?

6. What part of 6 are ⅔?

7. What part of ⅔ is 6?

8. What part of ⅔ are ⅔?

9. What are ⅔ of 8¼? ⅓ of 4⅔?

10. Reduce ⅔⅔ to a mixed number.

11. Reduce 7₁⁵₂ to an improper fraction.

12. Reduce ⅓⅔⅔ to its lowest terms.

13. What is ⅓ of ⅓? ⅓ of ¼?

14. Multiply ⅔ by 5. ⅔ by 3.

15. Multiply 5 by ⅔. 6 by 3⅔.

16. Multiply ⅔ by ⅔. 2½ by 3⅓.

17. Divide ⅓⅔ by 5. 6 by ⅔.

18. Divide 7 by ¼. 8 by 2⅔.

19. Divide ⅔ by ⅔. 2¼ by 3¼.

20. Add ½ and ⅓. ⅔ and ⅔.

21. Subtract ⅓ from ¼. ⅔ from ⅔.

22. ⅔ of 48 are ⅔ of what number?

23. John gathered 4⅔ bushels of walnuts, and William 5⅔ bushels; how many bushels did they both gather?

24. James had $1⅔, which he shared equally with 7 of his companions; how many cents did each receive?

25. A bought a coat for $12¼, a hat for $2⅔, and a pair of shoes for $4¼; how much change should he receive from a 20-dollar bill?

26. Robert is ⅔ as old as his mother, and ½ as old as his father; required the age of each, if ⅔ of Robert's age is 15 years.

27. A traveled 42½ miles in 8½ hours; how far did he travel in 1 hour?

28. Gilbert has 8⅔ quarts of chestnuts, and Louis has 9⅔ quarts; how many would each have if they shared them equally?

29. If my age be increased by the difference between its ¾ and ⅔, I will be 3 years older; required my age.

30. After spending ⅔ of my money, and ½ of the remainder, I had $10¼; how much had I at first?

31. Said E to F, "Your age is 36 years, and ⅔ of your age are ⅔ of what mine was 6 years ago;" required E's age.

32. M said to N, "Your age is 32 years, and ⅔ of your age are 4 years less than ⅔ of mine;" required M's age.

33. B sold his horse for ⅔ of the cost, and lost $50; what would he have gained by selling the horse for $216?

VI. DENOMINATE NUMBERS.

A **Denominate Number** is one in which the unit is a measure; as, 2 feet, 3 bushels, 5 pounds, etc.

I. MEASURES OF VALUE.

Money is the measure of the value of things. It is of two kinds, *Coin* and *Paper Money*.

NOTE.—For United States Money see page 30.

English Money.

English Money is the currency of Great Britain. The *unit* is the pound sterling.

TABLE.

4 farthings (far.)	=	1 penny, d.
12 pence	=	1 shilling, s.
20 shillings	=	1 pound, £.

EQUIVALENTS: £1 = 20 s. = 240 d. = 960 far.

The following denominations are also used:

5 shillings	=	1 crown.
21 shillings	=	1 guinea.

The approximate values in United States Money are:

£1 = $4.8665.   1 s. = 24 cts.   1 d. = 2 cts.

1. How many farthings in 3 pence? In 5 d.? In 7 d.?

2. How many pence in 12 farthings? In 20 far.? In 36 far.?

3. How many pence in 2 shillings? In 4 s.? In 7 s.?

4. How many shillings in 24 pence? In 60 d.? In 108 d.?

5. How many shillings in £5? In £7? In £20?

6. How many pounds in 40 s.? In 100 s.? In 160 s.?

7. How many pence in 5 s. 6 d.? In 8 s. 6 d.? In 10 s. 10 d.?

8. How many shillings in £5 6 s.? In £10 8 s.? In 5 crowns? In 5 guineas?

NOTES.—In *French Money* the unit of value is the *franc*, whose value is 19.3 cents.

In *German Money* the unit of value is the *mark* (Reichsmark), whose value is 23.85 cents.

II. MEASURES OF WEIGHT.

Avoirdupois Weight.

1. *Avoirdupois Weight* is used in weighing everything except precious metals, jewels, and some drugs.

TABLE.

16 ounces (oz.)	=	1 pound,	lb.
100 pounds	=	1 hundredweight,	cwt.
20 hundredweight	=	1 ton,	T.

EQUIVALENTS: 1 T. = 20 cwt. = 2000 lb. = 32,000 oz.

NOTE.—The avoirdupois pound contains 7000 grains.

1. How many ounces in 2 lb.? In 4 lb.? In 5½ lb.?

2. How many pounds in 48 oz.? In 80 oz.? In 96 oz.?

3. How many pounds in 2 T.? In 5 T.? In 10 T.?

4. How many tons in 4000 lb.? In 7000 lb.? In 60 cwt.?

5. What will 2½ pounds of cheese cost, at ¾ of a cent an ounce?

6. What will 3 T. 5 cwt. of hay cost, at $16 a ton?

7. What will 3 cwt. 20 lb. of coffee cost, at 30 cents a pound?

8. What will 8 pounds of sugar cost, if 8 oz. cost 2¼ cents?

Troy Weight.

Troy Weight is used in weighing precious metals and jewels.

TABLE.

24 grains (gr.)	=	1 pennyweight,	pwt.
20 pennyweights	=	1 ounce,	oz.
12 ounces	=	1 pound,	lb.

EQUIVALENTS: 1 lb. = 12 oz. = 240 pwt. = 5760 gr.

1. How many grains in 3 pwt.? In 5 pwt.? In 8 pwt.?

2. How many pennyweights in 48 gr.? In 96 gr.?

3. How many pennyweights in 5 oz.? In 6 oz.?

4. How many ounces in 32 pwt.?   In 50 pwt.?
5. How many ounces in 7 lb.?   In 12 lb.?
6. How many pounds in 42 oz.?   In 66 oz.?
7. How many ounces in 4 lb. 4 oz.?
8. How many pennyweights in 6 oz. 8 pwt.?

Apothecaries' Weight.

Apothecaries' Weight is used in mixing medicines.

TABLE.

20 grains (gr.)	=	1 scruple, ℈.
3 scruples	=	1 dram, ℨ.
8 drams	=	1 ounce, ℥.
12 ounces	=	1 pound, ℔.

EQUIVALENTS: 1 ℔. = 12℥ = 96 ℨ = 288 ℈ = 5760 gr.

The grain, ounce, and pound in apothecaries' weight are the same as in Troy weight.   Medicines are bought by avoirdupois weight, and are mixed and sold by apothecaries' weight.

III. MEASURES OF LENGTH.

Long Measure.

Long Measure is used for measuring lengths and distances.

TABLE.

12 inches (in.)	=	1 foot, ft.
3 feet	=	1 yard, yd.
5½ yards	=	1 rod, rd.
320 rods	=	1 mile, mi.

EQUIVALENTS: 1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

1. How many inches in 2 ft.?   In 5 ft.?   In 7 ft.?   In 3 yd. 2 ft.?
2. How many feet in 36 in.?   In 54 in.?   In 5 yd. 2 ft.?   In 4 rd.?
3. How many yards in 60 ft.?   In 4 rd. 4 yd.?
4. How many rods in 44 yd.?   In 66 yd.?
5. How many rods in 3 mi.?   In 4 mi.?
6. How many miles in 640 rd.?   In 960 rd.?
7. How many feet in a mile?   In ½ mi.?
8. How many rods in ½ mi.?   In ¼ mi.?

5

9. What part of a rod is 1 yd. ? 2 yd. ?

10. How many inches in 6¼ ft. ? In 5½ ft. ?

IV. MEASURES OF SURFACE.

Square Measure.

Square Measure is used in measuring surfaces.

TABLE.

144 square inches (sq. in.)	=	1 square foot, sq. ft.
9 square feet	=	1 square yard, sq. yd.
30¼ square yards	=	1 square rod, sq. rd.
160 square rods	=	1 acre, A.
640 acres	=	1 square mile, sq. mi.

EQUIVALENTS : 1 A. = 160 sq. rd. or P. (perches) = 4840 sq. yd. = 43,560 sq. ft. = 6,272,640 sq. in.

NOTE.—The area of a rectangular surface, as a door, is equal to the length multiplied by the width.

1. How many square inches in 2 sq. ft. ? In 4 sq. ft. ?

2. How many square feet in 10 sq. rd. ? In 12 sq. rd. ?

3. How many square yards in 36 sq. ft. ? In 72 sq. ft. ? In 108 sq. ft. ?

4. How many square rods in 121 sq. yd. ? In 363 sq. yd. ?

5. How many square rods in 2 acres ? In 2½ acres ? In 4¼ acres ?

6. A room is 6 ft. long and 5 ft. wide ; how many square feet in its area?

7. What is the area of a school-yard 4 rd. wide and 5 rd. long ?

8. How many acres in a field 20 rd. wide and 80 rd. long ?

9. What will it cost to cover a room 12 ft. long and 9 ft. wide with oilcloth, at 40 cents a square yard ?

V. MEASURES OF VOLUME.

Cubic Measure.

Cubic Measure is used for measuring solids and solid contents.

TABLE.

1728 cubic inches (cu. in.)	=	1 cubic foot, cu. ft.
27 cubic feet	=	1 cubic yard, cu. yd.

EQUIVALENTS : 1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

A *cord of wood* is a pile 8 ft. long, 4 ft. wide, and 4 ft. high, and therefore contains 128 cu. ft. ?

A *perch of stone* or of *masonry* is 16½ ft. long, 1½ ft. wide, and 1 ft. high, and contains 24¾ cu. ft.

A cubic yard of earth is called a *load*.

1. How many cubic inches in 2 cu. ft. ? In 5 cu. ft. ?

2. How many cubic feet in 3 cu. yd. ? In 10 cu. yd. ?

3. How many cubic feet in 2 cords? In 5 cords?

4. How many cubic feet in 4 perches of stone?

NOTE.—The volume of a rectangular solid is equal to the product of its three dimensions.

5. How many cubic inches in a block of marble 6 in. long, 4 in. wide, and 3 in. high?

6. How many cubic feet of water in a cistern 8 ft. long, 5 ft. wide, and 4 ft. deep?

7. What is the difference between a 3-inch cube and 3 cubic inches?

8. How many loads of earth in a cellar 27 ft. long, 20 ft. wide, and 4 ft. deep?

VI. MEASURES OF CAPACITY.

Liquid Measure.

Liquid Measure is used for measuring liquids.

TABLE.

4 gills (gi.)	=	1 pint, pt.
2 pints	=	1 quart, qt.
4 quarts	=	1 gallon, gal.

EQUIVALENTS : 1 gal. = 4 qt. = 8 pt. = 32 gi.

A liquid gallon contains 231 cu. in.

1. How many gills in 6 pt. ? In 8 pt. ? In 10½ pt. ?

2. How many pints in 16 gi. ? In 8 qt. ? In 16 qt. ?

3. How many quarts in 36 pt.? In 5 gal. ? In 40 gal. ?

4. Reduce 10 gal. 1 qt. to pints.

5. Reduce 1 qt. 1 pt. 1 gi. to gills.

6. What will 5 gal. of vinegar cost, at 3 cents a quart?

7. What will 5 gal. of oil cost, at 10 cents a quart?

8. A boy sold 10 gallons of milk, at 3 cents a pint; how much did he receive?

Dry Measure.

Dry Measure is used for measuring grain, fruit, vegetables, etc.

TABLE.

2 pints (pt.)	=	1 quart, qt.
8 quarts	=	1 peck, pk.
4 pecks	=	1 bushel, bu.

EQUIVALENTS: 1 bu. = 4 pk. = 32 qt. = 64 pt.

A bushel contains 2150.42 cu. in.

1. How many quarts in 3 pk.? In 6 pk.? In 8½ pk.?

2. How many pecks in 5 bu.? In 8 bu.? In 10 bu.? In 16 qt.? In 40 qt.?

3. How many bushels in 20 pk.? In 32 pk.? In 60 pk.? In 80 pk.?

4. Reduce 5 qt. 1 pt. to pints. 2 bu. to pints.

5. Reduce 3 bu. 3 pk. 3 qt. to quarts.

6. Reduce 47 pt. to pecks. 100 pt. to pecks.

7. How many quarts in a bushel? How many pints?

8. How much will 2½ bu. of chestnuts cost, at 10 cents a quart?

9. When peas are selling at 15 cents a half-peck, how much is that a bushel?

10. When apples are selling at 30 cents a peck, what is a barrel containing 3 bushels worth?

11. What is the cost of 2½ pecks of beans, at 80 cents a bushel?

12. What will 2 bushels of seed cost, at 5 cents a quart?

VII. MEASURES OF TIME.

TABLE.

60 seconds (sec.)	=	1 minute, min.
60 minutes	=	1 hour, hr.
24 hours	=	1 day, da.
7 days	=	1 week, wk.
365 days	=	1 common year, yr.
366 days	=	1 leap year, l. yr.
100 years	=	1 century, C.

NOTE.—In business transactions and in calculating interest, it is customary to regard 30 days in a month and 12 months in a year.

A. M. denotes before noon; M., noon; and P. M., afternoon.

Every year divisible by 4, except the centennial years, and every centennial year divisible by 400, is a leap year; all other years are common years.

The number of days in each month can be remembered by the following lines:

> Thirty days have September,
> April, June, and November.
> All the rest have thirty-one,
> Excepting February alone;
> To this we twenty-eight assign,
> Till leap year gives it twenty-nine.

1. How many seconds in 5 min.? In 10 min.? In 15 min.? In 30 min.?

2. How many minutes in 6 hr.? In 8 hr.? In 10 hr.? In 360 sec.?

3. How many hours in 3 da.? In 5 da.? In 10 da.? In 960 min.?

4. How many days in 7 wk.? In 10 wk.? In 12 wk.? In 144 hr.?

5. Reduce 8 mo. 24 da. to mo.

SOLUTION.—In 1 month there are 30 days; hence, 1 day is $\frac{1}{30}$ of a month, and 24 days are 24 times $\frac{1}{30}$, which are $\frac{24}{30}$, or $\frac{4}{5}$, of a month, which, added to 8 months, equal $8\frac{4}{5}$, or $\frac{44}{5}$ months?

6. Reduce 1 yr. 9 mo. 20 da. to yr.

7. Reduce 4 yr. 4 mo. 4 da. to yr.

8. Reduce 10 yr. 10 mo. 10 da. to yr.

9. Reduce 6 yr. 6 mo. 18 da. to yr.

10. A promised to pay a note, dated June 1st, in 60 days; when was it due?

11. B promised to pay a note, dated May 6th, in 90 days; when did it fall due?

12. If a note is dated July 18th, and was paid Nov. 24th, how long was it unpaid?

13. If a note, dated March 4th, was paid June 2d, how long was it unpaid?

14. B promised to pay a note, dated Jan. 18, 1896, in 90 days; when did it fall due?

VIII. ANGULAR MEASURE.

Angular Measure is used in measuring arcs and angles.

TABLE.

60 seconds ($''$)	=	1 minute, $'$.
60 minutes	=	1 degree, $°$.
360 degrees	=	1 circumference, C.

EQUIVALENTS: 1 C. = 360° = 21,600′ = 1,296,000″.

1. How many seconds in 4′? In 3½′? In 5′?

2. How many minutes in 8°? In 10°? In 480″? In 720″? In 960″?

3. How many degrees in 420′? In 600′?

4. How many degrees in the circumference of a circle? In ½ the circumference? In ¼ the circumference?

5. What part of a circumference is an arc of 30°? Of 45°? Of 60°? Of 120°?

IX. MISCELLANEOUS TABLES.

NUMBERS.				PAPER.		
12 units	=	1 dozen.		24 sheets	=	1 quire.
12 dozen	=	1 gross.		20 quires	=	1 ream.
20 units	=	1 score.		480 sheets	=	1 ream.

Miscellaneous Problems.

1. How much will 15 eggs cost, at 32 cents a dozen?

2. How much will 8 quires of paper cost, at ½ cent a sheet?

3. What will 10 lead pencils cost, at $2.88 a gross?

4. What will 20 pens cost, at $2.16 a gross?

5. What will 16 quires of paper cost, at $3 a ream?

6. What will 6 quarts of seed cost, at $6.40 a bushel?

7. At 48 cents a dozen, what will 2½ scores of pencils cost?

8. How many half-pint bottles will 6 gallons of writing fluid fill?

9. What will 5 pecks of peas cost, at $1.60 a bushel?

10. What will 10 gallons of vinegar cost, at 2 cents a pint?

11. How many half-hour periods can be made from 5 A. M. to 6 P. M.?

12. At $1.80 a gallon, what will 3 quarts of oil cost?

13. How much will 6 tons 4 cwt. of coal cost, at $5.25 a ton?

14. Find the cost of 2 gal. 1 qt. of oil, at 40 cents a quart.

15. How much will 18 oz. of beef cost, at 12 cents a pound?

16. How much will 4 bu. 2 pk. 1 qt. of salt cost, at 2 cents a quart?

17. How many days in the first four months of 1896?

18. How many feet in 10 miles?

19. A boy gathered 1½ bushels of chestnuts, and sold them at 8 cents a quart; how much did he receive for them?

20. How many pens in 30 boxes, if each box contains ½ a gross?

21. When potatoes are selling at 25 cents a half-peck, what are they worth a bushel?

22. At 3 cents a pint, what is the milk bill for July of a family which uses 3 quarts each day?

23. What will 5 lb. of spice cost, at the rate of 3 cents an ounce?

24. A man bought 20 dozen eggs, at 35 cents a score; what did they cost him?

25. How many steps of 3 feet each will a person take in going 50 rods?

26. How many ink-wells, each containing ½ a gill, can be filled with 3 gallons of ink?

27. How many quart boxes will 5 bu. 3 pk. of strawberries fill?

28. When apples are selling at $1.60 a bushel, what are they worth a half-peck?

29. How many sheets of paper in 2 reams and 2½ quires?

30. What will 5 rods of fence cost, at the rate of $1.20 a foot?

31. How many square feet in a board 16 ft. long and 15 in. wide?

32. Find the cost of 3½ gross of marbles, at 4 cents a dozen.

33. Reduce 4 mo. 20 da. to the fraction of a year.

34. A merchant received $3 for a can of milk, at 3 cents a pint; how many gallons did it hold?

35. Find the cost of 3 yd. 2 ft. 3 in. of silk, at $3.60 a yard.

36. How many days from March 22d to Sept. 3d?

37. A farmer had 3 bu. 2 pk. 5 qt. of seed, which he sold at 2 cents a pint; how much did he receive for it?

38. An agent bought 2½ dozen of books, at $8 a dozen; for what sum must he sell them to gain 25 cents apiece?

39. How many square feet in a board 12 ft. long and 12 in. wide?

40. How many square feet in a board 16 ft. long and 9 in. wide?

VII. PERCENTAGE.

Percentage is the process of computation in which the basis of operation is 100.

The term *Per Cent* means *by the hundred.*

The symbol % stands for *per cent.*

In the example, 6% of $200 = $\frac{6}{100}$ of $200 = $12. The *base* is $200, the *rate per cent* is 6%, or $\frac{6}{100}$, and the *percentage* is $12.

I. To Express Per Cent as a Fraction.

1. What fractional part of a number is 20% of it?

SOLUTION.—20% of a number is $\frac{20}{100}$, or $\frac{1}{5}$, of the number. Therefore, etc.

What fractional part of a number is—

2. 5% ?	*6.* 15% ?	*10.* 30% ?	*14.* 48% ?
3. 8% ?	*7.* 20% ?	*11.* 35% ?	*15.* 50% ?
4. 10% ?	*8.* 25% ?	*12.* 40% ?	*16.* 60% ?
5. 12% ?	*9.* 28% ?	*13.* 45% ?	*17.* 75% ?

What fractional part of a number are—

18. $6\frac{1}{4}$% ?	*22.* $18\frac{3}{4}$% ?	*26.* $28\frac{4}{7}$% ?	*30.* $33\frac{1}{3}$% ?
19. $8\frac{1}{3}$% ?	*23.* $13\frac{1}{3}$% ?	*27.* $31\frac{1}{4}$% ?	*31.* $66\frac{2}{3}$% ?
20. $12\frac{1}{2}$% ?	*24.* $14\frac{2}{7}$% ?	*28.* $37\frac{1}{2}$% ?	*32.* $62\frac{1}{2}$% ?
21. $16\frac{2}{3}$% ?	*25.* $26\frac{2}{3}$% ?	*29.* $43\frac{3}{4}$% ?	*33.* $87\frac{1}{2}$% ?

II. To Express a Fraction as a Per Cent.

1. What per cent of a number is $\frac{1}{4}$ of it?

SOLUTION.—A number is 100 per cent of itself, and $\frac{1}{4}$ of the number is $\frac{1}{4}$ of 100%, or 25%. Therefore, etc.

OR IN PRACTICE.—$\frac{1}{4} = \frac{25}{100}$, or 25%.

What per cent of a number—

2. Is $\frac{1}{2}$ of it?	*5.* Is $\frac{1}{6}$ of it?	*8.* Is $\frac{1}{9}$ of it?
3. Is $\frac{1}{3}$ of it?	*6.* Is $\frac{1}{7}$ of it?	*9.* Is $\frac{1}{10}$ of it?
4. Is $\frac{1}{5}$ of it?	*7.* Is $\frac{1}{8}$ of it?	*10.* Is $\frac{1}{11}$ of it?

11. Is $\frac{1}{12}$ of it?
12. Is $\frac{1}{15}$ of it?
13. Is $\frac{1}{40}$ of it?
14. Is $\frac{1}{25}$ of it?
15. Is $\frac{1}{50}$ of it?
16. Are $\frac{3}{4}$ of it?
17. Are $\frac{3}{4}$ of it?

18. Are $\frac{2}{3}$ of it?
19. Are $\frac{5}{8}$ of it?
20. Are $\frac{3}{10}$ of it?
21. Are $\frac{3}{50}$ of it?
22. Are $\frac{2}{25}$ of it?
23. Are $\frac{3}{25}$ of it?
24. Are $\frac{7}{20}$ of it?

25. Are $\frac{9}{20}$ of it?
26. Are $\frac{5}{16}$ of it?
27. Are $\frac{9}{16}$ of it?
28. Are $\frac{3}{4}$ of it?
29. Are $\frac{5}{8}$ of it?
30. Are $\frac{1}{4}$ of it?
31. Is $\frac{1}{16}$ of it?

III. To Find the Percentage.

1. What is 5% of 40?

SOLUTION.—5% of a number is $\frac{5}{100}$, or $\frac{1}{20}$, of it. $\frac{1}{20}$ of 40 is 2. Therefore, etc.

What is—

2. 10% of 60?
3. 6% of 50?
4. 20% of 30?
5. 25% of 40?

6. 40% of 25?
7. 15% of 120?
8. 8% of 75?
9. 24% of 125?

10. 16% of 75?
11. 25% of 85?
12. 5% of 150?
13. 6% of 200?

14. What is 60% of 60? 70% of 70? 80% of 80?
15. What are $12\frac{1}{2}$% of 48? $33\frac{1}{3}$% of 72? $66\frac{2}{3}$% of 24?
16. What are $6\frac{1}{4}$% of 48? $16\frac{2}{3}$% of 66? $6\frac{2}{3}$% of 45?
17. What are $11\frac{1}{9}$% of 81? $12\frac{1}{2}$% of 32? $14\frac{2}{7}$% of 49?
18. What are $8\frac{1}{3}$% of 144? $9\frac{1}{11}$% of 77? $37\frac{1}{2}$% of 40?
19. What are $62\frac{1}{2}$% of 64? $83\frac{1}{3}$% of 72? $85\frac{5}{7}$% of 28?
20. What are $18\frac{3}{4}$% of 32? $88\frac{2}{3}$% of 80? $56\frac{1}{4}$% of 64?

21. A bought a horse for $250, and sold him at a gain of 10%; what was the gain?

SOLUTION.—At a gain of 10%, $\frac{10}{100}$, or $\frac{1}{10}$, of the cost equal the gain. $\frac{1}{10}$ of $250 is $25. Therefore, etc.

22. B bought a cow for $40, and sold her at a loss of 10%; what was the loss?

23. C bought a boat for $15, and sold it at a gain of 20%; what was the gain?

24. D earned $500 a year, and spent 25% of it for books; how much had he remaining?

SUGGESTION.—100% − 25% = 75%, or $\frac{3}{4}$, of $500.

25. E had a farm of 80 acres, and sold 40% of it; how many acres had he remaining?

26. A farmer raised 200 bushels of wheat, and sold 25% of it to A, and 15% to B; how much had he remaining?

27. Mrs. Brown bought a bill of goods to the amount of $120, and was allowed a discount of 10% for cash; how much did she pay?

28. A carriage was bought for $160, and sold at a loss of 25%; required the selling price.

29. James had $50, and spent 20% of it for a coat, and 20% of the remainder for a vest; how much had he still remaining?

30. If a house costs $2500, at what price must it be sold to gain 20%? At what price, to lose 10%?

31. If eggs are bought at 30 cents a dozen, at what price must they be sold to gain 16⅔%?

32. A sold his horse, which cost him $200, to B, at a gain of 25%; and B sold him to C, at a loss of 25%; did C pay more or less for the horse than A, and how much?

33. 37½% of $160 are ⅔ of what James paid for a watch; required the cost of the watch.

34. A merchant gained $500, and invested 60% of it in tea at 75 cents a pound; how many pounds did he buy?

35. 10% of $300 is ⅔ of what Harold paid for a bicycle. He afterward sold it at a loss of 25%; how much did he receive for it?

36. A began business with $1200. The first year he gained 16⅔% of his capital, the second year he gained 14⅔% of what he had at the end of the first year, and the third year he lost 25% of what he had at the end of the second year; how much money had he then?

IV. To Find the Rate.

1. A bought a gun for $10, and sold it for $8; what was the loss per cent?

SOLUTION.—If A bought a gun for $10, and sold it for $8, he lost the difference between $10 and $8, which is $2. If he lost $2 on $10, he lost $\frac{2}{10}$, or ⅕, of the cost, or 20%.

2. A student paid $4 for a book, and sold it for $6; what was the gain per cent?

3. Henry bought a violin for $16, and sold it for $20; what was the gain per cent?

4. B bought a carriage for $80, and sold it for $70; what was the loss per cent?

5. A boy carried 2½ dozen eggs to a store, but, falling, broke 3 of them; what per cent was broken?

6. A merchant bought lead pencils at 12 cents a dozen, and sold them at 3 cents apiece; what was the gain per cent?

SUGGESTION.—The gain per cent is the same if bought at 1 cent apiece and sold at 3 cents apiece as if bought at 12 cents a dozen and sold at 36 cents a dozen.

7. A boy bought apples at 10 cents a score, and sold them at 2 cents apiece; what was the gain per cent?

8. A bought oranges at the rate of 5 for 4 cents, and sold them at the rate of 4 for 5 cents; what was the gain per cent? $\; \;$

9. If ⅔ of a quantity of grain are sold for ⅔ of the cost of the entire quantity, what is the gain or loss per cent?

10. B bought 10 cows for $400, and sold 8 of them for what they all cost; what was the gain per cent on those sold?

11. A bought a wagon for $80, and sold it to B at a gain of 20%; what per cent would B have lost if he had sold it for $80?

12. If ⅔ of an article are sold for ½ of its value, what is the gain per cent?

13. By selling eggs at 16 cents a dozen, I gained 33⅓%; what per cent would I have gained by selling them at 30 cents a score?

14. ⅔ of 30 are what per cent of ½ of 80?

15. ⅚ of 42 are what per cent of ⅔ of 40?

16. ⅚ of 72 are what per cent of ₁⁹₀ of 120?

17. A teacher's salary is $1500 a year. He spends $300 for boarding, $200 for traveling, and $250 for books; what per cent of his salary does he spend for each, and what per cent does he save?

18. John sold his horse for $80, and thereby lost $16; what was the loss per cent?

19. William sold his watch for $120, and thereby gained $24; what was the gain per cent?

20. A bought 12 barrels of sugar for $150, and sold 33⅓% of it for $62½; what was the gain per cent on what he sold?

21. ½ of 30% is what per cent of 40%?

22. ⅘ of 40% are what per cent of 87½%?

23. ⅖ of 45% are what per cent of ¾ of 162%?

24. If lemons are bought at the rate of 3 for 2 cents, and sold at the rate of 2 for 3 cents, what is the gain per cent?

25. Amos sold 60% of his sheep for 80% of the cost of all; what was the gain per cent on those sold?

26. If 1½ quarts of salt are mixed with 8 gallons of water, what per cent of the mixture is salt?

27. A man sold a horse for $250, and thereby gained 25%; what per cent would he have lost if he had sold him for $175?

28. A had 12 gallons of cider, and sold 6 quarts; what per cent of it did he sell?

29. If 2 quarts of water are added to 4½ gallons of wine, what per cent of the mixture is water?

30. A father, leaving a fortune of $10,000, gave his son 45% of it, and the remainder to his daughter; what per cent of the son's share did the daughter receive?

31. A drover bought 60 sheep, at $5 a head; 6 of them having died, he sold the remainder at $6 a head; what per cent did he gain on the whole transaction?

V. To Find the Base.

1. A sold his bicycle for $20 more than cost, and thus gained 25%; what was the cost?

SOLUTION.—By the conditions of the problem, $20 are 25%, or ¼, of the cost. If $20 are ¼ of the cost, the cost is 4 times $20, which are $80. Therefore, etc.

2. A sold his house at a gain of $300, and thereby gained 10%; what was the cost of the house?

3. A merchant sold cloth at a gain of 25 cents a yard, which was a gain of 12½%; required the cost of the cloth.

4. A farmer sold a horse for $250, which was at a loss of 33⅓%; what did the horse cost him?

5. A miller sold flour at $6 a barrel, which was at a gain of 11½%; required the cost of the flour.

6. 20 is 2% of what number?   5%?

7. 24 is 3% of what number?   16⅔%?

8. 18 is $\frac{1}{3}\%$ of what number? $\frac{1}{3}\%$?

9. 30 is $28\frac{4}{7}\%$ of what number? $37\frac{1}{2}\%$?

10. C sold his watch at a loss of $16\frac{2}{3}\%$, and thereby lost $15; what did the watch cost?

11. James sold his bicycle for $120, thereby losing $11\frac{1}{9}\%$; how should he have sold it so as to gain $11\frac{1}{9}\%$?

12. If by selling a carriage for $130 I lost $18\frac{3}{4}\%$, what should I have received for it to have gained $12\frac{1}{2}\%$?

13. A house was sold for $2600, which was at a loss of 35%; how should it have been sold to have gained 35%?

14. William sold a watch for $110, and thereby lost 12%; what per cent would he have gained by selling it for $140?

15. A bought a horse, and, after paying $12 for his keeping, sold him for $162, losing 40% on the transaction; how should he have sold him in order to gain 20%?

16. If I gain 15% by selling a carriage for $184, what per cent would I have lost by selling it for $150?

17. A drover sold 2 horses for $120 a head; on one he gained 25%, and on the other he lost 25%; how much did he gain or lose by the transaction?

18. B sold two boats for $60 each; on one he gained 25%, and on the other he lost $16\frac{2}{3}\%$; how much did he gain or lose by the transaction?

19. M sold N a carriage for $200, and gained 25%; N then sold it, and lost 20%; how much more did N lose than M gained?

20. A man sold 2 bicycles for $80 each; on one he lost 20%, and on the other he gained 25%; did he gain or lose by the transaction, and how much?

21. An organ was sold for $30 less than its value, which was at a loss of 20%; what per cent would have been gained if it had been sold for $175?

22. A sold a horse for $150, and thereby lost 25%; he then bought another for $150, and sold him at a gain of 25%; what was the gain or loss on the entire transaction?

23. B sold his horse and carriage for $450, receiving for the horse 2 times as much as for the carriage; on the horse he lost 10%, and on the carriage he gained 25%; did he gain or lose on the entire transaction, and how much?

VI. Miscellaneous Problems.

1. What is 20% of 80?

2. 80 is 20% of what number?

3. What per cent of 80 is 20?

4. If the gain equals ¼ of the cost, what is the gain per cent?

5. If the gain equals ¼ of the selling price, what is the gain per cent?

6. If ¾ of the gain equal ½ of the cost, what is the gain per cent?

7. If ¾ of the gain equal ½ of the selling price, what is the gain per cent?

8. What is the gain per cent when the selling price is twice the cost?

9. What is the loss per cent when the cost is twice the selling price?

10. What is the gain per cent when 2½ times the gain equal the selling price?

11. What is the gain per cent when ½ of the selling price equals the gain?

12. What is the loss per cent when 5 times the loss equal the cost?

13. What is the loss per cent when ½ of the selling price equals the loss?

14. What is the loss per cent when ½ of the loss equals the selling price?

15. What is the loss per cent when 3 times the selling price equal the loss?

16. What is the loss per cent when 3 times the loss equal the selling price?

17. What is the gain per cent when ¾ of the selling price equal 3¼ times the gain?

18. What is the loss per cent when ¾ of the selling price equal 3¼ times the loss?

19. If I buy goods at 20% below cost, and sell them at 20% above cost, what per cent do I gain?

20. A tailor bought 10 coats for $120, and sold 8 of them for what they all cost; what was the gain per cent on those sold?

21. A merchant makes a reduction of 10% for cash; how much is A's bill, who pays $54?

22. A invested 60% of his money in wheat, and had $300 remaining; how much did he invest in wheat?

23. A farmer sold sheep at an average price of $12 per head, and thus lost 25 % ; what per cent would he have gained if he had sold them at $18 per head?·

. *24.* What must I ask for a boat which cost $60, so that, after falling 14⅔% from my asking price, I may still gain 20 % ?

25. Mr. Lansinger bought a horse for $250, and set such a price on him that, after falling $50, he still gained 40% ; what per cent did he abate from the asking price?

26. The number of girls in a certain school is 40 % of the number of boys; what per cent of the number of girls is the number of boys?

27. Mr. Bowman sold a carriage for $240, and thereby cleared ⅕ of this amount; what per cent would he have gained if he had sold it for $280?

28. 60 is 25 % less, and 25 % more, than what numbers?

29. A drover lost 25% of his sheep; what per cent must he gain on the remainder in order to gain 30% on the cost of all?

30. A merchant sold a watch and chain for $168, receiving for the chain ⅔ as much as for the watch; on the watch he gained 14⅔%, and on the chain he lost 25 % ; did he gain or lose on the entire transaction, and how much?

31. By selling melons at 18 cents each, Robert cleared ⅕ of the cost; what per cent would he have lost by selling them at 14 cents each?

32. A bought a bookcase for $60, which was 20% less than cost, and sold it at 20% more than cost; required the gain per cent.

33. B reduced the price of his carriage $15, and thereby decreased his gain from 20% to 15% ; what was the cost of the carriage?

34. A merchant marked goods at 25% above cost, and sold them at a reduction of 25% from the marked price, supposing that he was thus selling his goods at cost, but afterward found that he had lost $5; required the cost of the goods.

35. William bought a lot of pencils at 2 cents apiece, and twice as many at 3 cents each ; what was his gain per cent if he sold them all at 40 cents a dozen?

36. Walter sold two horses for $420 each, gaining 40% on the one, and losing 40% on the other; what was his gain or loss per cent on the transaction?

VIII. APPLICATIONS OF PERCENTAGE.

I. COMMISSION.

1. AN agent sold $500 worth of goods at 2% commission; required his commission.

SUGGESTION.—His commission is 2% of $500.

2. An agent charges 3% for collecting bills; required his commission for collecting $600.

3. An agent sold a quantity of wheat on a commission of 2%; what was the agent's commission if the owner received $980?

4. My agent received $780 to invest in cotton, after deducting his commission of 4%; required his commission.

5. An agent sold 80 barrels of flour at $5 a barrel, charging 2½% commission; required his commission.

6. An agent bought 40 horses at $150 a head, charging 3% commission; required his commission.

7. An agent receives $210 to purchase goods, after deducting his commission, which is 5% on the amount invested; required his commission.

8. A receives 4% for purchasing sugar; how much does he invest in sugar if his commission is $24?

9. B sold grain on a commission of 2½%; how much did his employer receive if B's commission was $60?

10. A house was sold which yielded the net proceeds of $3760, after deducting $40 expenses and the auctioneer's commission of 5%; for what sum was the house sold?

11. An agent receives $40 for selling $800 worth of goods; required his commission.

12. An agent received $120 for selling goods which yielded his employer $4680; required the agent's per cent of commission.

13. M sent his agent $420 to invest in cattle at $20 a head; how many cattle did he buy if his commission was 5%?

14. An agent sold $1000 worth of cotton at 5% commission. He invested the net proceeds in sugar at $10 a barrel, after deducting his commission of 5⅜% for buying; how many barrels of sugar did he buy?

15. An agent sold 40 cows at $30 a head, on a commission of 10%. He invested the proceeds in flour at $5 a barrel, after deducting his commission of 8%; how many barrels of flour did he buy?

·*16.* I sold goods on a commission of 5%, through an agent who charged me 3%; my commission after paying my agent was $80; what sum is due my employer?

17. What is the rate of commission when $\frac{1}{12}$ of the proceeds equals the commission?

18. How much grain must a farmer take to the mill, that he may bring away the flour of 18 bushels after the miller has taken 10% toll?

II. Trade Discount.

It is customary among wholesale dealers to allow their customers several discounts; as, " 10 and 5 off," meaning 10% off, and 5% off the remainder. Sometimes the reduction is designated as " ¼ and 10 off," meaning ¼ off the list price and 10% off the remainder.

List price, retail price, and *marked price* are generally used as synonymous terms. The *net price* is the amount paid for the goods.

1. What is due on a bill of $600, subject to a discount of 10 and 5% off?

SOLUTION.—At a discount of 10%, $\frac{10}{100}$, or $\frac{1}{10}$, of the cost, equal the discount: $\frac{1}{10}$ of $600 is $60; and the proceeds are $600 minus $60, or $540. At a discount of 5%, $\frac{5}{100}$, or $\frac{1}{20}$, of the remainder equal the second discount; $\frac{1}{20}$ of $540 is $27; and the net proceeds are $540 minus $27, or $513. Therefore, etc.

2. Find the amount due on a bill of $400, subject to a discount of 10 and 4% off.

3. A merchant marked an article $100, but gave a discount of 20 and 10% off; how much did he receive for it?

4. What is due on a bill of $600, subject to a discount of 20 and 5% off?

5. A merchant marks goods 40% above cost, and sells at a discount of 25% from the list price; what is his gain per cent?

6

6. What must a merchant ask for an article which cost $60, that he may give a discount of 10% and still gain 20% ?

7. What reduction from the price of an article is 20 and 10% off?

8. What is the difference between a discount of 50%, and one of 25 and 20% off?

9. At what per cent above cost must a merchant mark goods, that he may sell at 16⅔% less than list price and still gain 15% ? ·

10. A bought a watch for $120, and set such a price on it that, after falling $16, he still gained 20% in the sale of it; what per cent did he abate from his asking price?

III. INSURANCE.

The *premium* is the sum paid for insurance. The *policy* is the written agreement between the insurance company and the person insured.

1. What is the premium for insuring $2000 on a house, at 1% ?

SOLUTION.—At 1%, $\frac{1}{100}$ of the amount insured equals the premium; $\frac{1}{100}$ of $2000 is $20. Therefore, etc.

2. What is the premium for insuring a house for $5000, at 1½% a year?

3. A insured his store, worth $6000, for ¾ of its value, at 1% a year; what was the premium?

4. A house worth $9000 was insured for ⅝ of its value, at ⅔% ; what was the premium?

5. A merchant insured a cargo worth $8000 for ¾ of its value, at 3½% ; in case of shipwreck, what would be his actual loss?

6. A paid $25 for insuring $2500 worth of goods; what was the rate of insurance?

7. If I pay a premium of $90 for insuring my house, worth $4800, at ¾ of its value, what is the rate of insurance?

8. A paid a premium of $100 for insuring his store, worth $10,000, at 80% of its value; what was the rate of insurance?

9. For what sum must a house, worth $5600, be insured at the rate of 2%, so that, in case of loss, the owner may receive both the premium and ⅞ of the value of the house?

10. A man insured his house for ⅘ of its value, at a 2½% rate. The house was burned, and his loss, including the insurance premium, was $880; what was the value of the house?

11. A mill is insured for ¾ of its value, at 1½% ; what is the value of the mill if the premium is $36?

12. B's store was insured for $\frac{9}{10}$ of its value, at 2½%. The building was destroyed by fire, and his loss, including the premium paid, was $490; what was the value of the store?

IV. STOCKS.

If the par value is not mentioned, it is understood to be $100 a share.

When stock is quoted at 120, it means that $100 worth of stock is selling at $120, or at 20% premium; when it is quoted at 90, it means that $100 worth of stock is selling for $90, or at a discount of 10%.

1. A owns 40 shares of stock in a company that declares a 5% dividend; required his dividend.

SOLUTION.—Since the dividend is 5%, the income from 1 share is $5; and the income from 40 shares is 40 times $5, or $200.

2. B owns 30 shares of bank stock, and receives a dividend of 6% ; what is his dividend?

3. C received $60 as his share of a 5% dividend; how many shares did he own?

4. D owns 60 shares of railroad stock, and receives a 5½% dividend; what is his dividend? '

5. Mr. Brown receives $450 from his investment in 5% stock; how many shares does he own?

6. If 50 shares of bank stock yield an annual dividend of $300, what is the rate of income?

SOLUTION.—If 50 shares yield $300, 1 share will yield $\frac{1}{50}$ of $300, or $6; that is, $6 on $100, or 6%.

7. If 90 shares of stock yield an annual income of $360, what is the rate of income?

8. What rate of dividend is declared on stock, when 120 shares yield an annual income of $660?

9. What will 10 shares of stock cost, bought at 15% premium?

10. What will 20 shares of bank stock cost, bought at 10% discount?

11. When stock is selling at 10% premium, how many shares can be bought for $4400?

12. What is the rate of dividend, when 15 shares of bank stock gain $75?

13. B bought stock at 97 and sold it at 103, and thereby gained $600; how many shares had he?

14. A merchant bought stock at 101 and was obliged to sell at 98, thereby losing $360; how many shares had he?

NOTE.—*Brokerage* is a commission charged by a broker for buying or selling stocks or bonds, and is always estimated on the *par value.*

15. Find the cost of 10 shares of railroad stock, bought at 119½, brokerage ½%.

SOLUTION.—One share costs $119½ + $½ brokerage, or $120; and 10 shares will cost 10 times $120, which are $1200. Therefore, etc.

16. A bought 20 shares of bank stock, at 110½, through a broker who charged him ½% brokerage; required the cost of the stock.

17. Mr. Noble sold 40 shares of railroad stock, at 112¼, through an agent who charged him ¼% brokerage; how much did he receive?

18. B bought railroad stocks, at 3½% discount, and sold them at 2½% premium, through a broker who charged him ½% for both buying and selling; how many shares did he buy, if he gained $60?

V. INTEREST.

Interest is money paid for the use of money.

The **Principal** is the sum for which interest is charged.

The **Amount** is the sum of the principal and interest.

The **Rate of Interest** is the interest on one dollar for one year.

NOTE.—In interest, a year is 12 mo. of 30 da. each.

I. To Find the Interest.

GENERAL METHOD.

1. What is the interest of $80 for one year at 5%?

SOLUTION.—At 5%, $\frac{5}{100}$, or $\frac{1}{20}$, of the principal equal the interest. $\frac{1}{20}$ of $80 is $4. Therefore, etc.

What is the interest for one year—

2. Of $60 at 5%?	*5.* Of $300 at 7%?
3. Of $100 at 6%?	*6.* Of $40 at 10%?
4. Of $200 at 4%?	*7.* Of $50 at 8%?

. What is the interest for one year—

8. Of $400 at 4½%?	*13.* Of $500 at 5%?
9. Of $500 at 6%?	*14.* Of $200 at 7%?
10. Of $75 at 4%?	*15.* Of $400 at 5%?
11. Of $300 at 6%?	*16.* Of $500 at 8%?
12. Of $150 at 10%?	*17.* Of $600 at 9%?

18. What is the interest of $400 for 2 years at 5%?

SOLUTION.—For 2 years at 5%, $\frac{10}{100}$, or $\frac{1}{10}$, of the principal equal the interest. $\frac{1}{10}$ of $400 is $40. Therefore, etc.

What is the interest—

19. Of $60 for 4 yr. at 5%?	*27.* Of $500 for 10 yr. at 6%?
20. Of $40 for 5 yr. at 5%?	*28.* Of $440 for 11 yr. at 5%?
21. Of $125 for 6 yr. at 6%?	*29.* Of $800 for 6½ yr. at 6%?
22. Of $300 for 8 yr. at 5%?	*30.* Of $600 for 4 yr. at 5%?
23. Of $250 for 4 yr. at 7%?	*31.* Of $500 for 10 yr. at 5%?
24. Of $350 for 7 yr. at 6%?	*32.* Of $120 for 8 yr. at 5%?
25. Of $150 for 4 yr. at 4½%?	*33.* Of $350 for 5 yr. at 5%?
26. Of $225 for 8·yr. at 6%?	*34.* Of $900 for 6 yr. at 5%?

35. What is the interest of $200 for 4 yr. 6 mo. at 6%?

SOLUTION.—4 yr. 6 mo. equal 4½ yr. For 4½ yr. at 6%, $\frac{27}{100}$ of the principal equal the interest; $\frac{27}{100}$ of $200 equal $54. Therefore, etc.

What is the interest—

36. Of $275 for 2 yr. 4 mo. at 6%?

37. Of $450 for 3 yr. 3 mo. at 8%?

38. Of $484 for 4 yr. 2 mo. at 6%?

39. Of $600 for 5 yr. 9 mo. at 4%?

40. Of $400 for 4 yr. 3 mo. at 5%?

41. Of $550 for 3 yr. 8 mo. at 6%?

42. Of $420 for 5 yr. 5 mo. at 12%?

43. Of $600 for 6 yr. 6 mo. at 6%?

44. Of $750 for 3 yr. 10 mo. at 12%?

45. Of $600 for 7 yr. 2 mo. at 4%?

46. What is the interest of $300 for 4 yr. 4 mo. 15 da. at 8%?

SOLUTION.—4 mo. 15 da. equal 135 da. 135 da. equal $\frac{135}{360}$, or $\frac{3}{8}$, of a year, which, added to 4 yr., equal 4⅜ yr. For 4⅜ yr. at 8%, $\frac{35}{100}$, or $\frac{7}{20}$, of the principal equal the interest. $\frac{7}{20}$ of $300 equal $105. Therefore, etc.

What is the interest—

47. Of $500 for 7 yr. 2 mo. 12 da. at 5% ?

48. Of $600 for 4 yr. 9 mo. 18 da. at 5% ?

49. Of $250 for 4 yr. 10 mo. 20 da. at 9% ?

50. Of $400 for 6 yr. 4 mo. 24 da. at 10% ?

51. Of $425 for 4 yr. 5 mo. 10 da. at 4½% ?

52. Of $500 for 6 yr. 9 mo. 18 da. at 7½% ?

53. Of $400 for 5 yr. 7 mo. 15 da. at 6% ?

54. Of $500 for 2 yr. 3 mo. 27 da. at 8% ?

55. Of $600 for 6 yr. 6 mo. 6 da. at 6% ?

56. What is the amount of $500 for 4 yr. at 5% ?

SOLUTION.—For 4 yr. at 5%, $\frac{20}{100}$, or $\frac{1}{5}$, of the principal equal the interest, which, added to $\frac{5}{5}$ of the principal, equal $\frac{6}{5}$ of the principal, or the amount. $\frac{6}{5}$ of $500 are $600. Therefore, etc.

What is the amount—

57. Of $300 for 6 yr. at 5% ?

58. Of $400 for 4½ yr. at 8% ?

59. Of $500 for 5 yr. at 5% ?

60. Of $240 for 3 yr. 9 mo. at 4% ?

61. Of $600 for 5 yr. 8 mo. at 6% ?

62. Of $400 for 4 yr. 4 mo. at 9% ?

63. Of $350 for 8 yr. 3 mo. at 8% ?

64. Of $400 for 4 yr. 3 mo. 10 da. at 9% ?

65. Of $800 for 3 yr. 5 mo. 12 da. at 5% ?

66. Of $700 for 7 yr. 7 mo. 15 da. at 8% ?

67. Of $600 for 5 yr. 5 mo. 18 da. at 5% ?

68. A bought a house for $3000; at what sum must he rent it per month to realize 6% interest on the investment, if he pays $20 for taxes?

69. C built a house which cost him $2400; what does he pay annually for the use of it, if he borrows the money at 5½%, and pays $15 tax?

70. Which is the cheaper, and how much,—to rent a property which cost $2500, at $20 a month, or to buy it and borrow the money at 5½%, providing the taxes and other expenses are $22½?

71. $200 are ¼ of M's and ½ of N's money; what will be the amount of each for 4 yr. 6 mo., at 8%?

SIXTY-DAY METHOD.

Interest is so frequently computed for short periods of time as to render the following method of great service in business:

Since 60 days are $\frac{1}{6}$ of a year, the interest on $1 for 60 days at 6% is 1 cent. Therefore, the interest of any sum of money for 60 days at 6% can be found by simply moving the decimal point two places to the left.

Thus, the interest of $300 for 60 days at 6% is $3.00. For 63 days it is $3.00 plus $\frac{1}{20}$ of $3.00, or $3.15, since 3 days are $\frac{1}{20}$ of 60 days. For 90 days the interest is $1\frac{1}{2}$ times the interest for 60 days, etc.

What is the interest for 60 days at 6% —

1. Of $540?	*7.* Of $1000?
2. Of $120?	*8.* Of $1400?
3. Of $360?	*9.* Of $2050?
4. Of $724?	*10.* Of $3020?
5. Of $864?	*11.* Of $4500?
6. Of $125?	*12.* Of $5000?

What is the interest for 90 days at 6% —

13. Of $400?	*18.* Of $320?
14. Of $600?	*19.* Of $480?
15. Of $800?	*20.* Of $500?
16. Of $660?	*21.* Of $640?
17. Of $120?	*22.* Of $720?

23. What is the interest of $100 for 63 da. at 6%?
24. What is the interest of $120 for 72 da. at 6%?
25. What is the interest of $130 for 120 da. at 6%?
26. What is the interest of $600 for 60 da. at 5%?
27. What is the interest of $240 for 60 da. at 7%?
28. What is the interest of $360 for 40 da. at 6%?

II. To Find the Principal.

1. What principal will in 4 years, at 5%, give $80 interest?

SOLUTION.—For 4 yr. at 5%, $\frac{20}{100}$, or $\frac{1}{5}$, of the principal, equal the interest. If $\frac{1}{5}$ of the principal is $80, $\frac{5}{5}$ of the principal are 5 times $80, which are $400. Therefore, etc.

What principal will—

2. In 5 years, at 6%, give $60 interest?
3. In 6 years, at 7%, give $84 interest?

What principal will—

  4. In 8 years, at 5%, give $72 interest?

  5. In 7 years, at 4%, give $105 interest?

  6. In 5 yr. 6 mo., at 8%, give $220 interest?

  7. In 8 yr. 4 mo., at 6%, give $42.50 interest?

  8. In 4 yr. 9 mo., at 4%, give $380 interest?

  9. In 6 yr. 6 mo., at 6%, give $390 interest?

  10. A's annual income, from money on interest at 5%, is $450; how much money has he at interest?

  11. How much money can I borrow at 6%, if I pay an annual interest of $420?

  12. Mr. Smith bought a farm, paying one half in cash, and borrowing the remainder at 5½%; required the cost of the farm, if his annual interest is $682.

  13. How much money must be put on interest for 4 yr., at 6%, to give the same interest as $700 for 6 yr., at 8%?

  14. The interest of A's and B's money for 8 yr., at 5%, is $3400; what is the fortune of each, if ⅔ of A's money equal ¼ of B's?

  15. What principal will in 8 yr., at 5%, give $840 amount?

SOLUTION.—For 8 yr., at 5%, $\frac{40}{100}$, or ⅖, of the principal equal the interest. ⅖ of the principal plus ⅖ of the principal equal ⅞ of the principal, or the amount, which is $840. If ⅞ of the principal equal $840, ⅛ of the principal is ⅟ of $840, or $120; and ⅖, or the principal, equal 5 times $120, which are $600. Therefore, etc.

What principal will—

  16. In 10 yr., at 6%, give $1600 amount? .

  17. In 7 yr., at 8%, give $780 amount?

  18. In 6 yr., at 5%, give $520 amount?

  19. In 4 yr. 6 mo., at 6%, give $254 amount?

  20. In 5 yr. 3 mo., at 8%, give $213 amount?

  21. In 7 yr. 4 mo., at 9%, give $332 amount?

  22. In 9 yr. 9 mo., at 8%, give $712 amount?

  23. The amount of ⅔ of the cost of A's house for 8 years, at 5%, is $5600; required the cost of the house.

  24. The amount of A's and B's fortunes for 6 years, at 5%, is $10,400; what is the fortune of each, if A's equals ⅓ of B's?

25. The amount of ⅔ of the cost of a farm, for 10 years at 5%, is $6000; required the cost of the farm.

26. The amount of M's and N's money, for 4 years at 5%, is $2160; how much has each, if ⅔ of M's equal ⅘ of N's?

27. If the amount of ⅔ of the cost of A's carriage, for 10 years at 5%, be put on interest for the same time and rate, it will amount to $450; required the cost of the carriage.

III. To Find the Time.

1. In what time will $400, at 5%, give $60 interest?

SOLUTION.—At 5% for 1 year, $\frac{5}{100}$, or $\frac{1}{20}$, of the principal equal the interest. $\frac{1}{20}$ of $400 is $20. If in 1 year $400 give $20 interest, to give $60 interest it will require as many years as 20 is contained times in 60, which are 3, or 3 years. Therefore, etc.

In what time—

2. Will $100, at 4%, give $16 interest?

3. Will $200, at 6%, give $60 interest?

4. Will $300, at 7%, give $84 interest?

5. Will $400, at 5½%, give $66 interest?

6. Will $500, at 8%, give $120 interest?

7. Will $600, at 6%, give $144 interest?

8. Will $120, at 5%, give $72 interest?

9. Will $150, at 6%, give $54 interest?

10. Will $300, at 8%, give $60 interest?

11. Will $400, at 5%, give $65 interest?

12. Will $600, at 7%, give $740 amount?

13. Will $800, at 4%, give $872 amount?

14. Will $300, at 5%, give $350 amount?

15. In what time will a principal double itself at 5%? 6%? 8%? 10%?

16. In what time will a principal treble itself at 4%? 5%? 10%? 12½%?

17. The amount of a certain principal, for a certain time at 6%, is $672; and, for the same time at 8%, it is $696; required the principal and time.

18. The amount of a certain principal, for a certain time at 6%, is $650; and, for the same time at 6½%, it is $662.50; required the principal and time.

19. A certain sum of money on interest amounts, in a certain time at 6%, to $780; and at 8%, for twice the time, to $1080; required the principal and times.

20. A certain sum of money on interest amounts, in a certain time at 5%, to $480; and at 6%, for thrice the time, to $688; required the principal and times.

IV. To Find the Rate.

1. At what per cent will $40 gain $12 in 5 years?

SOLUTION.—For 5 years at 1%, $\frac{5}{100}$, or $\frac{1}{20}$, of the principal equal the interest. $\frac{1}{20}$ of $40 is $2. If $40 at 1% gain $2, to gain $12 it will require as many per cent as 2 is contained times in 12, which are 6, or 6%. Therefore, etc.

At what per cent—

2. Will $200 gain $40 in 5 years?
3. Will $300 gain $90 in 6 years?
4. Will $400 gain $140 in 7 years?
5. Will $500 gain $220 in 8 years?
6. Will $160 gain $112 in 7 years?
7. Will $80 gain $48 in 12 years?
8. Will $250 gain $80 in 4 years?
9. Will $80 amount to $96 in 5 years?
10. Will $600 amount to $726 in 3½ years?
11. Will $500 amount to $640 in 7 years?
12. Will $400 amount to $505 in 5 yr. 3 mo.?
13. Will $300 amount to $378 in 4 yr. 4 mo.?
14. Will $600 amount to $817 in 5 yr. 2 mo.?

15. At what per cent will a principal double itself in 20 yr.?   10 yr.?   12½ yr.?

16. At what per cent will a principal treble itself in 20 yr.?   33⅓ yr.?   40 yr.?

17. At what per cent will a principal quadruple itself in 30 yr.?   60 yr.?   50 yr.?

18. The amount of a certain principal, for 6 years at a certain per cent, is $520, and the amount, at the same rate for 8 years, is $560; required the principal and rate per cent.

19. The amount of a certain principal, for 8 years at a certain per cent, is $700, and the amount, at the same rate for 6 years, is $650; required the principal and rate per cent.

20. The amount of a certain principal, for 7 years at a certain per cent, is $426, and the amount, at twice that rate for 4 years, is $444; required the principal and the first rate per cent.

V. Present Worth.

The **Present Worth** of a sum of money due at some future time, without interest, is such a sum as being on interest for the time will amount to the given sum.

The **True Discount** is the difference between the given sum and the present worth.

The *present worth* corresponds to the *principal*, the *discount* to the *interest*, and the given *sum* to the *amount*.

1. What is the present worth of $600, due 4 years hence, at 5% ?

SOLUTION.—For 4 years, at 5%, $\frac{20}{100}$, or $\frac{1}{5}$, of the present worth equal the interest, which, added to $\frac{5}{5}$ of the present worth, equals $\frac{6}{5}$ of the present worth, or $600; hence, the present worth is $\frac{5}{6}$ of $600, or $500. Therefore, etc.

Find the present worth—

    2. Of $620, due 4 years hence, at 6%.

    3. Of $490, due 8 years hence, at 5%.

    4. Of $640, due 5 years hence, at 5%.

    5. Of $381, due 4 yr. 6 mo. hence, at 6%.

    6. Of $484, due 5 yr. 3 mo. hence, at 4%.

Find the true discount—

    7. Of $650, due 3 yr. 4 mo. hence, at 9%.

    8. Of $670, due 5 yr. 8 mo. hence, at 6%.

    9. Of $800, due 2 yr. 6 mo. hence, at 10%.

    10. Of $360, due 7 yr. 4 mo. hence, at 6%.

11. A bought goods to the amount of $406, on a credit of 3 mo.; what sum in cash will pay the debt, money being worth 6% ?

12. B bought a piano on 1 yr. 6 mo. credit; but, by paying cash, received a discount of $60; required the cost of the piano, money being worth 8%.

VI. Bank Discount.

Bank Discount is a sum of money charged by a bank for the payment of a note before it becomes due.

It is the simple interest on the face of the note for the given time.

NOTE.—In some States three "days of grace" are allowed.

The difference between the face of a note and the discount is the **Proceeds.**

1. What is the bank discount of $200 for 60 days at 6%?

SOLUTION.—Since the bank discount is the same as the interest, the bank discount of $200 for 60 days is $2.00; or for 63 days, $2.10.

Find the bank discount of—

2. $300 for 60 days at 6%.	*8.* $800 for 90 days at 9%.
3. $220 for 90 days at 6%.	*9.* $400 for 75 days at 6%.
4. $400 for 30 days at 6%.	*10.* $700 for 60 days at 6%.
6. $600 for 90 days at 5%.	*11.* $300 for 30 days at 6%.
5. $600 for 60 days at 8%.	*12.* $1200 for 90 days at 5%.
7. $800 for 30 days at 7%.	*13.* $1000 for 60 days at 6%.

14. Find the proceeds of a 60-day note for $300, discounted at 5%.

15. What must be the face of a note which, when discounted at a bank for 90 days at 8%, will yield $490 proceeds?

16. What must be the face of a note which, when discounted at a bank for 60 days at 6%, will yield $990 proceeds?

17. A bank bought a 90-day note for $30 less than its face; required the face if discounted at 6%.

18. Find the proceeds of a 90-day note for $800, discounted at 6%.

19. What must be the face of a note which, when discounted at a bank for 60 days at 9%, will yield $1970 proceeds?

20. What must be the face of a note which, when discounted at a bank for 30 days at 6%, will yield $995 proceeds?

21. A bank bought a 60-day note for $90 less than its face; required the face if discounted at 9%.

22. For what sum must I give my note for 60 days, so that when discounted at 6% it will yield $2970?

23. For what sum must I give my note for 90 days, so that when discounted at 8% it will yield $392?

24. Find the proceeds of a 30-day note for $1200, discounted at 6%.

IX. PROPORTION.

I. PARTITIVE PROPORTION.

1. DIVIDE $60 between A and B so that their shares shall be to each other as 2 to 3.

SOLUTION.—Since the shares are to be to each other as 2 to 3, we must divide $60 into 2 + 3, or 5, equal parts. 2 of these parts, or ⅖ of $60, equal $24, which are A's share; and 3 of these parts, or ⅗ of $60, equal $36, which are B's share. Therefore, etc.

2. Divide the number 36 into two parts that shall be to each other as 4 to 5.

3. Divide 50 cents between B and C so that their shares shall be to each other as 4 to 6.

4. Divide the number 90 into two parts that shall be to each other as 4 to 5.

5. Divide 64 marbles between John and Ralph so that John shall have 7 as often as Ralph has 9.

6. In a school of 1050 pupils, there are 7 girls to every 8 boys; how many of each sex in the school?

7. Divide 240 into three parts that shall be in the proportion of 1, 2, and 3.

8. In a certain field there are 66 animals, consisting of horses, sheep, and cows; how many are there of each kind, if the horses, sheep, and cows are in the proportion of 1, 2, and 3?

9. The sum of two numbers is 60, and the first is to the second as ½ to ⅓; what are the numbers?

10. A and B contribute $1000 toward building a church, which is to be situated 2 miles from A and 3 miles from B; how much should each contribute, if they pay in proportion to the reciprocals of their distances?

SUGGESTION.—The reciprocal of a number is 1 divided by that number.

11. A furnished 5 loaves for a meal, and B 7 loaves, while C contributed 20 cents; how should the money be divided between A and B?

12. A has 90 acres of land, and B has 60 acres; C pays them $600 for an interest in the crop; how should the money be divided between A and B, if they all share the crop equally?

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13. A earned ⅔ as much money as he already had, and then had $50; how much did he have at first?

SOLUTION.—After earning ⅔ as much as he had, he had ⅔ + ⅓, or ⅗, of his money, which equal $50. If ⅗ of his money are $50, his money must be ⅕ of $50, or $30. Therefore, etc.

14. B spent ¾ of his money for a watch, and had $48 remaining; how much had he at first?

15. A's money, being increased by its ₁₄₅, equals $5700; how much did he have at first?

16. M's age, if increased by its ½ and ⅔, would be 26 years; how old is he?

17. A dog having killed 5 of B's sheep, B bought 15 more and then had ¾ as many as at first; how many sheep had he at first?

18. A pole 60 feet long was broken into two unequal pieces, so that the length of the shorter piece was ⅔ of the length of the longer; required the length of each piece.

19. An army, after losing ⅓ of its number in battle, received a reinforcement of 16,000 men, and then had ¾ as many as at first; required the number of men in the army before the battle.

20. James, after spending ⅘ of his money, found that $40 were ⅝ of what he had remaining; how much money had he at first?

21. Emerson lost ⅝ of his money, then spent ½ as much as he lost, and yet had $70 remaining; how much had he at first?

22. ¼ of the length of a pole is in the air, ⅔ are in the water, and 14 feet in the ground; required the length of the pole.

23. A borrowed ⅔ of M's money, and, after spending ⅘ of it, returned the remainder, which was $90 less than he spent; required M's money.

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24. Three times a number, increased by 10, equal 28; what is the number?

SOLUTION.—If 3 times a number, increased by 10, are 28, then 3 times the number are 28 minus 10, or 18, and the number is ⅓ of 18, or 6. Therefore, etc.

25. B earned twice as much money as he had, and, after finding $12 more, had $72; how much had he at first?

26. If 20 be added to 3 times a certain number, the result will be 80; required the number.

27. ⅔ of A's age, increased by 7 years, equal 27 years; how old is he?

28. If 30 be added to 5 times a certain number, the result will be 110; what is the number?

29. A farmer sold ⅔ of his sheep, and a dog killed 4 of the remainder; how many sheep had he at first, if he then had ⅓ of the number remaining?

30. A horse and carriage cost $250; required the cost of each if ⅔ of the cost of the horse, minus $12, equal $120.

31. A flag pole is 130 feet high, and ⅔ of its height are 8 feet more than ⅔ of the length of its shadow; required the length of its shadow.

32. A watch and chain cost $140, and $\frac{7}{10}$ of their cost lack $32 of being 1⅔ times the cost of the watch; required the cost of the chain.

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33. If 5 times a number, diminished by 60, equal 2 times the number, what is the number?

SOLUTION.—If 5 times a number, diminished by 60, equal 2 times the number, then 60 must equal the difference between 5 times the number and 2 times the number, or 3 times the number. If 3 times the number are 60, the number is ⅓ of 60, or 20. Therefore, etc.

34. M's age, increased by 50 years, equals 3 times his age; what is M's age?

35. What is the price of butter when 8 pounds cost 64 cents more than 6 pounds?

36. What is the price of eggs per dozen when 4½ dozen cost 36 cents more than 3 dozen?

37. Four times a number, plus 8, equal 3 times the number, plus 12; what is the number?

38. Six times a number, minus 8, equal 7 times the number, minus 20; what is the number?

39. Five times a number, minus 10, equal 2 times the number, plus 20; what is the number?

40. A farmer sold 20 bushels more than ⅜ of his corn, and had ⅘ of it remaining; how many bushels had he at first? ⁻ .

41. ₁/₁₀ of the number of pupils in a school, diminished by 35, equal ⅜ of the number; how many pupils in the school? - ⌐⟩

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42. M and N together have $50; how much has each, if N has $10 more than M?

SOLUTION.—Once M's money plus $10, which equal N's, added to M's money, equal 2 times M's money plus $10, or what both have, which are $50. Then 2 times M's money are $50 — $10, or $40, and M's money is ½ of $40, or $20. Therefore, etc.

43. John has 12 marbles more than William, and they together have 60; how many marbles has each?

44. The sum of two numbers is 120, and their difference is 40; what are the numbers?

45. A and B had each the same sum of money; after A earned $12 and B spent $8, they together had $52; how much had each at first?

46. Two drovers had each the same number of cattle; one lost 12, and the other 8; how many had each at first, if they together now have 70?

47. A and B had each the same number of sheep; A sold 10 of his, and then bought 12 from B; how many had each after this change, if they together now have 100?

48. B walked 90 miles in three consecutive days: how far did he walk each day, if he walked 5 miles less each day than on the preceding day?

49. C walked 120 miles in four consecutive days; how far did he walk each day, if he walked 8 miles more each day than on the next preceding day?

50. D walked 90 miles in five consecutive days; how far did he walk each day, if he walked 4 miles less each day than on the next preceding day?

NOTE.—The course presented here is thought to be sufficiently full for the needs of most schools. But teachers who desire a more extended . course will find a number of Supplementary Problems beginning on page 113. The examples for this section are the problems from 1 to 10 inclusive.

II. COMPOUND PROPORTION.

1. If 6 men earn $72 in 3 days, how much will 7 men earn in 5 days?

SOLUTION.—If 6 men earn $72 in 3 days, 1 man will earn ⅙ of $72, or $12, in 3 days; and in 1 day 1 man will earn ⅓ of $12, or $4. If 1 man earns $4 in 1 day, 7 men will earn 7 times $4, or $28, in 1 day; and in 5 days they will earn 5 times $28, or $140. Therefore, etc.

2. If 5 men can do a piece of work in 8 days of 10 hours each, in how many days of 8 hours each can 10 men do the same work?

3. How long will 8 tons of hay last 12 horses, if 6 tons last 5 horses 24 weeks?

4. How long will 12 barrels of flour last 15 persons, if 5 barrels last 3 persons 20 weeks?

5. If 6 men build 5 rods of wall in 10 weeks, how many men will be required to build 15 rods in 6 weeks?

6. If 6 loaves of bread cost 25 cents when flour is worth $5 a barrel, what will 10 loaves cost when flour is worth $6 a barrel?

7. If 8 oxen eat 3 acres of grass in 12 weeks, how many acres will 4 oxen eat in 16 weeks?

8. If 6 men can do a piece of work in 12 days, in what time will it be completed if 3 men leave when the work is half done?

9. If 6 men, in 16 days of 10 hours each, dig a trench 8 rods long, in how many days of 8 hours each can 8 men dig a trench 6 rods long?

NOTE.—For additional examples see Supplementary Problems, 11 to 20 inclusive, page 114.

III. PARTNERSHIP.

1. A and B in partnership gain $2200: A put in $5000, and B $6000; what is each man's share of the gain?

SOLUTION.—Since A put in $5000 and B $6000, the gain must be divided in the proportion of 5000 to 6000, or as 5 to 6. If we divide $2200 into 5 + 6, or 11, equal parts, 5 of these parts, or $\frac{5}{11}$ of $2200, equal $1000, A's share; and 6 of these parts, or $\frac{6}{11}$ of $2200, equal $1200, B's share of the gain. Therefore, etc.

2. M and N formed a partnership: M put in $2000, and N $3000; they gained $450; what was each man's share of the gain?

3. E and F engaged in business with a capital of $12,000, of which E contributed $5000, and F the remainder; they gained $2400; what was each man's share of the gain?

4. B and C engaged in business: B put in 4 times as much capital as C; if they gained $1500, what was each man's share of the gain?

5. A, B, and C engaged in business: A put in ⅔ as much as B, and B ⅔ as much as C; if they gained $950, what was each man's share of the gain? •

6. A and B enter into partnership, and gain $400: A owns ¾ of the stock lacking $100, and his share of the gain is $250; how much stock does each own?

7. M and N in partnership gain $600: M owns ⅔ of the stock, plus $120, and his share of the gain is $420; how much stock does each own?

. *8.* R and S bought a farm, and sold it at a gain of $1500; how much did the farm cost, if R owned ½ of it plus $500, and his share of the gain was $550? | ⅄

9. John plowed ½ of a field and 5 acres more, and received $14; if the total cost of plowing the field was $27, how large was the field?

10. P and Q built a wall for $960: P built ⅔ of the wall, plus 12 rods, and received $504; how long is the wall?

11. A and B hired a pasture for $48: A pastured 4 cows, and B 2 cows; how much should each pay?

12. A, B, and C hired a pasture for $63: A put in 5 cows, B 7, and C 9; how much should each pay?

13. M and N agree to do a piece of work for $120: M sends 12 men, and N 16 boys; how much should each receive, if a man does as mu.h work as 2 boys?

14. Two men hired a pasture for $64: the first put in 11 horses, and the second 14 cows; how much should each pay, if 2 cows eat as much as 3 horses?

15. A and B agreed to dig a trench for $156: A sent 6 men for 5 days, and B 8 men for 6 days; how much should each receive?

16. A, B, and C hired a pasture for $54: A pastured 8 cows for 4 weeks, B 12 horses for 6 weeks, and C 30 sheep for 8 weeks; how much should each pay, if a cow eats twice as much as a horse, and a horse three times as much as a sheep?

X. ALGEBRAIC EXERCISES.

1. EXPRESS the sum of a and b.   *Ans.*, $a + b$.   Express the difference of a and b.   *Ans.*, $a - b$.

2. Express the sum of x and y; of c and d; of m and n.

3. Express the difference of x and y; of c and d.

4. In the expressions 3 times 2, and 5 times x, or $5x$, what numbers are the multipliers?

5. What is meant by $4x$? $8a$? $9b$?

6. What is meant by $\frac{a}{b}$? $\frac{x}{y}$? $\frac{m}{n}$?

7. Shorten the expression $x + x + x + x$.

8. Shorten the expression $3x + 2x + x$.

9. Read the equations $12 = 4 + 8$; $16 = 12 + 4$; $x = 4 + 5$; $x = 14 - 5$.

10. Find the value of x in each of the following equations:

11. $2x = 12$.

12. $5x = 20$.

13. $7x = 21$.

14. $3x + 2x = 15$.

15. $7x - 3x = 16$.

16. $10x - 4x = 36$.

17. $\frac{x}{2} = 8$.

18. $\frac{x}{3} = 5$.

19. $\frac{2x}{7} = 6$.

20. $x + \frac{x}{2} = 12$.

21. $x + \frac{2x}{3} = 5$.

22. $\frac{2x}{3} + \frac{x}{4} = 11$.

23. $\frac{3x}{4} + \frac{x}{4} = 5$.

24. $x - \frac{3x}{5} = 10$.

25. A bought a horse and a cow for $300; what was the cost of each, if the horse cost 5 times as much as the cow?

Let $x =$ the cost of the cow.
Then, $5x =$ the cost of the horse.
And $6x = \$300$, the cost of both.
Whence, $x = \$50$, the cost of the cow.
And $5x = \$250$, the cost of the horse.

26. The sum of two numbers is 60; what are the numbers, if the first is five times the second?

27. A number increased by its $\frac{1}{4}$ is 80; what is the number?

28. B's age diminished by its $\frac{1}{4}$ is 30 years; how old is B?

29. If $x + 5 = 17$, what is the value of x?

30. If $4x = 3x + 10$, what is the value of x?

NOTE.—In transposing a quantity from one side of an equation to the other, we change its sign.

Find the value of x in each of the following equations:

31. $3x + 4 = 16.$

32. $5x - 2 = 8.$

33. $4x + 9 = 21.$

34. $3x + 5 = 2x + 8.$

35. $7x - 4 = 3x + 8.$

36. $x + 4 = \dfrac{x}{2} + 6.$

37. $x - 5 = \dfrac{2x}{3} + 4.$

38. $x + 5 = \frac{1}{3}(x + 45).$

39. $5x - 8 = 3x + 4.$

40. $9x + 10 = 7x + 30.$

41. Five times a number, plus 6, equal 3 times the number, plus 12; what is the number?

42. A horse and carriage cost $320; what was the cost of each, if the horse cost twice as much as the carriage, minus $40?

43. If A's age be increased by its $\frac{3}{4}$ and 9 years more, it will equal 30 years; what is his age?

44. M and N together have $60; how much has each, if M has $12 more than N?

45. Robert's age, increased by 40 years, equals 5 times his age; required his age.

46. Six times a number, minus 6, equal 7 times the number, minus 14; required the number.

47. E, being asked his age, said, "Three times my age 5 years ago equal two times my age 5 years hence;" what is his age?

48. If 30 be added to 4 times a certain number, the result will be 150; what is the number?

49. The difference between $\frac{1}{5}$ of a number and $\frac{1}{10}$ of the number is 60; what is the number?

50. A and B together have $240; how much has each, if A has 7 times as much as B?

XI. INVOLUTION AND EVOLUTION.

I. Involution.

Involution is the process of finding any power of a number.

The **Power** of a number is the product obtained by using the number several times as a factor.

The **Square** of a number is the product obtained by using the number twice as a factor, as $4 \times 4 = 16$.

The **Cube** of a number is the product obtained by using the number three times as a factor, as $3 \times 3 \times 3 = 27$.

1. Square 1, 2, 3, 4, 5, 6, 7, 8, 9.

2. Square 10, 11, 12, 13, 14, 15, 16, 17.

3. Square 18, 19, 20, 21, 22, 23, 24, 25.

4. Cube 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

5. Square $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{2}{3}$.

6. Cube $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{3}$.

II. Evolution.

Evolution is the process of finding a root of a number.

A **Root** of a number is one of the equal factors of the number.

The **Square Root** of a number is one of its two equal factors. Thus, the square root of 25 is 5, since 5×5 are 25.

The **Cube Root** of a number is one of its three equal factors. Thus, the cube root of 125 is 5, since $5 \times 5 \times 5$ are 125.

1. Find the square root of 4, 9, 16, 25, 36, 49, 64, 81, 100.

2. Find the cube root of 8, 27, 64, 125, 216, 343, 512, 729.

3. The square of twice a number is 144; what is the number?

4. The square of 3 times a number is 36; what is the number?

5. $\frac{2}{3}$ of the square of a number are 54; what is the number?

6. $\frac{1}{4}$ of the cube of a number are 100; what is the number?

7. The square of $\frac{2}{3}$ of a number is 64; what is the number?

8. The cube of $\frac{1}{2}$ of a number is 27; what is the number?

9. One half of the square of a number is 128; what is the number?

10. One half of a number, multiplied by ¼ of the number, is 24; what is the number?

11. The difference between the square of twice a number and the square of the number is 108; what is the number?

12. The difference between the square of twice a number and the square of one half of the number is 60; what is the number?

13. Fifty is ¼ of the difference between the square of twice a number and twice the square of a number; required the number.

14. Three times the square of a number are 27 more than 9 times the square of half the number; what is the number?

15. Three fourths of the square of a number are 12 more than the square of ¾ of the number; required the number.

16. One half of the cube root of a number is 5; what is the number?

17. ⅔ of the cube of a number are 80 more than the cube of ⅓ of the number; required the number.

18. One fourth of a number, multiplied by ⅕ of the number, is 45; required the number.

19. Five sixths of the square root of a number, multiplied by ⅗ of the square root of the number, are 80; required the number.

20. One third of a number, multiplied by ¼ of the number, is 33 less than the square of that number; required the number.

21. A field is 18 rods long, and 8 rods wide; what is the length of one side of a square field of equal area?

22. How much more fence would be required for a lot 24½ rods long, and 8 rods wide, than for a square lot of equal area?

23. A box is 9 feet long, 6 feet wide, and 4 feet high; required the side of a cubical box of equal contents.

24. A block of marble is 16 inches long, 8 inches wide, and 4 inches thick; how much more surface does it have than a cubical block of equal contents?

25. A man traveled ½ of the distance from Philadelphia to New York the first day, ⅓ of the remainder the second day, and ½ of what now remained the third day, and then found that he was still 26¾ miles from New York; required the distance from Philadelphia to *New York.*

XII. PROBLEMS FOR ANALYSIS.

1. IF a man can do a piece of work in 6 days, how much of it can he do in 5 days?

SOLUTION.—If a man can do a piece of work in 6 days, in 1 day he can do ⅙ of it, and in 5 days he can do 5 times ⅙, or ⅚, of it. Therefore, etc.

2. A can mow a field in 7 days; how much of it can he mow in 4 days? ⁴⁄₇ ·~·⸴

3. A can mow an acre of grass in ½ day; how much can he mow in 5 days?

4. B can build a boat in 5 days; what part of it can he build in ¾ of a day? ⸴ ⸴ · ·

5. C can do ₁⁵₂ of a piece of work in a day; in what time can he do the whole work?

6. M can do a piece of work in 6 days, and N in 8 days; how much can they together do in a day? ⁷⸴ ⸴ ⸴

7. A can build a wall in 4 days, and B in 6 days; in what time can they build it, working together? ⸴

8. B can do a piece of work in 2 days, C in 3 days, and D in 4 days; in what time can they together do it? ⸴ ⸴ ⸴

9. M can do a piece of work in 3 days, N in 4 days, and P in 6 days; in what time will they do it, working together?

10. A can do a piece of work in ⅔ of a day, B in ¾ of a day, and C in 4⁄9 of a day; in what time can they do it, working together?

11. C can reap ⅔ of a field in a day, and D can reap the field in ¾ of a day; in what time can both reap it, working together? · ·

12. A cistern has two pipes: by the first it can be filled in 8 hours, and by the second in 12 hours; in what time can both fill it?

NOTE.—For additional examples see Supplementary Problems, 21 to 30 inclusive, pages 114 and 115.

13. How far can a person ride in a car going at the rate of 30 miles an hour, and return by carriage at the rate of 10 miles an hour, if he is gone but 8 hours?

SOLUTION.—If he goes 30 miles an hour, to go 1 mile will require $\frac{1}{30}$ of an hour; if he returns at the rate of 10 miles an hour, to return 1 mile will require $\frac{1}{10}$ of an hour; hence, to go and return 1 mile will require $\frac{1}{30}$ + $\frac{1}{10}$, or $\frac{4}{30}$, of an hour. He can go and return as many miles as $\frac{4}{30}$ are contained times in 8, or 60 miles. Therefore, etc.

14. How far may a person ride on a bicycle going at the rate of 12 miles an hour, and be gone but 9 hours, provided he returns on a car at the rate of 36 miles an hour?

15. How far may a person ride on a trolley going at the rate of 18 miles an hour, provided he is gone but 11 hours, and walks back at the rate of 4 miles an hour?

16. How far may I sail in a vessel going at the rate of 20 miles an hour, provided I am gone but 7 hours, and return at the rate of 15 miles an hour?'

17. A boat, whose rate of sailing in still water is 12 miles an hour, sails down a stream whose current is 4 miles an hour; how far may it go that it shall be gone but 6 hours?

18. Suppose that for every 5 cows a farmer has, he should plow an acre of land, and allow 1 acre of pasture for every 3 cows; how many cows could he keep on 32 acres of land?

19. Suppose that for every 4 cows a farmer has, he should plow an acre of land, and allow 1 acre of pasture for every 2 cows; how many cows could he keep on 15 acres? How many acres would he have if he kept 15 cows?

20. A boat, whose rate of sailing in still water is 15 miles an hour, sails down a stream whose current is 3 miles an hour; how far may it go that it shall both go and return to the place from which it started in 10 hours?

21. How far may I sail in a vessel going at the rate of 18 miles an hour, provided I am gone but 14 hours, and return at the rate of 24 miles an hour?

NOTE.—For additional examples see Supplementary Problems, 31 to 40 inclusive, pages 115 and 116.

22. A boy bought oranges at 2 cents each, and had 18 cents remaining; if he had paid 5 cents each, he would have had no money remaining; how many oranges did he buy?

SOLUTION.—By the second condition of the problem he gave, 5 minus 2, or 3, cents more for each orange than by the first condition, and for all of them 18 cents more; hence, he bought as many oranges as 3 is contained times in 18, or 6 oranges. Therefore, etc.

23. A drover bought sheep at $5 each, and had $43 remaining; had he paid $7 each, he would have had $3 remaining; how many sheep did he buy?

24. A farmer bought ducks at 35 cents each, and had $2 remaining; had he bought the same number of chickens at 40 cents apiece, he would have had 80 cents remaining; how many ducks did he buy?

25. A girl bought ribbon at 8 cents a yard, and had 60 cents remaining; had she bought ribbon at 9 cents a yard, she would have had 6 cents remaining; how many yards of ribbon did she buy?

26. A teacher gave her pupils 3 cents each, and had 80 cents remaining; had she given them 8 cents each, she would have lacked 80 cents of having money enough to pay them all; how many pupils had she?

27. A farmer, wishing to buy some cows, found that, if he bought those at $32 each, he would have $12 remaining; but if he bought those at $35 each, he would lack $12 of having money enough to pay for them; how many cows did he wish to buy?

28. A boy paid 60 cents for an equal number of apples and oranges, giving 2 cents each for the apples, and 3 cents each for the oranges; how many of each kind did he buy?

29. A drover bought an equal number of sheep and cows for $210, giving $6 each for the sheep, and $24 each for the cows; how many of each kind did he buy?

30. A farmer bought sheep at $6 apiece, and had $20 remaining; had he paid $8 apiece, he would have lacked $20 of having money enough to pay for them; how much money had he?

NOTE.—For additional examples see Supplementary Problems, 41 to 45 inclusive, page 116.

XIII. REVIEW OF PERCENTAGE.

1. IF a quart of water is added to 6 gallons of wine, what per cent of the mixture is water?

2. I sold a watch for $60, thereby gaining 20%; what per cent would I have gained by selling it for $72?

3. A merchant bought lead pencils at 6 cents a dozen, and sold them at 2 cents apiece; what was the gain per cent?

4. B sold lead pencils at 3½ cents apiece, and thereby gained 600%; what did they cost per dozen?

5. A bought a number of eggs at 1½ cents each, and 3 times as many at 1¼ cents each; what per cent did he gain, if he sold them all at the rate of 4 for 6 cents?

6. M sold N a watch, and gained 20%; N sold it to P for $48, and lost 20%; what did M pay for it?

7. B bought stocks at 40% below par, and sold them at 40% above par; how much did he invest, if he gained $800?

8. What must I ask for a horse which cost me $120, that I may drop 25% in price and still gain 20% on the cost?

9. What must I ask for a piano which cost me $400, that I may make a reduction of 20 and 25% off and still gain 50%?

10. If I buy buttons at 9 cents a dozen, and sell them at 80 cents a score, what per cent do I gain?

11. If an article had cost 10% less, and had been sold for the same price as marked, the gain would have been 66⅔%; required the first gain per cent.

12. If A's horse had cost him 20% less, and had been sold for the same price as he received, the gain would have been 75%; required the first gain per cent.

13. If the selling price is 1½ times the loss, what is the loss per cent?

14. If the selling price is 4 times the gain, what is the gain per cent?

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15. If the cost equals ⅝ of the gain, what is the gain per cent?

16. If the cost equals ⅛ of the gain, what is the gain per cent?

17. A collector receives 5% for collecting a school tax: his returns are $5700; how much was assessed, if he is able to collect 93¾% of the assessment?

18. A insured his factory for ⅞ of its real value, at 4% per annum: it was burned down, and his loss, including the insurance, was $2000; required the cost of the factory.

19. A company insured a frame store for ¾ of its real value, at 5% per annum: the store was destroyed by fire, and the company's loss was $11,400; required the value of the store.

20. B sold his house to A: if it had cost B 10% more, and had been sold for the same sum, the gain would have been 20%; required the original gain per cent.

21. If I sell ¾ of an article for what ½ of it cost me, what per cent do I lose on the part sold?

22. If ¾ of the cost equal the selling price, what is the loss per cent?

23. If ¾ of the selling price equal the cost, what is the gain per cent?

24. At what must I buy 6% stocks to realize 5% on my investment?

25. If I realize 6% income from my investment in 4½% bonds, what do they cost me?

26. ¾ of A's money equal ⅓ of B's; and the interest for 2 yr. 6 mo., at 6%, of ½ of A's and ⅓ of B's money is $36; required the amount possessed by each.

27. A sold a horse and carriage for $450: on the horse he gained 33⅓%, and on the carriage he lost 25%; did he gain or lose on the transaction, if ¾ of the cost of the horse equal ¾ of the cost of the carriage?

28. A tailor increased the price of a suit of clothes $10, and thereby increased his profit from 25% to 50%; what was the first marked price?

29. A offered his horse for sale, but, finding no purchaser at the amount asked, fell in price $20, and thereby diminished his gain from 33⅓% to 25%; required the first price asked.

30. C offered his watch for sale at a gain of 25%, but, finding no purchaser at the amount asked, fell in price $30, thereby losing 25%; required the cost of the watch.

31. A bought a horse for $240, and set such a price on him that, after a reduction of $32, he still gained 20% on the cost; what per cent did he abate from the asking price?

32. James sold a carriage at a loss of 20%: had the carriage cost $60 less, he would have gained 10%; required the selling price.

- *33.* A merchant sold a piano at a gain of 30%: had the piano cost $50 more, he would have gained only 10%; required the selling price.

34. A farmer sold two cows for $203: on the first he gained 25%, and on the second he lost 25%; did he gain or lose on the transaction, and how much, if the first cow cost 1¼ times as much as the second?

35. B has 3 times as much money as C, plus $60; and the interest for 2 yr. 6 mo., at 6%, of what they both have is $39; how much has each?

36. An agent received $1638 to invest in an equal number of sheep at $6, cows at $30, and horses at $120 each, after reserving his commission of 5% for buying; how many of each did he purchase?

37. A lady bought vinegar at 21 cents a gallon: 10% was lost by leakage; at what price per gallon must she sell the remainder to gain 20% on the cost of all?

38. A merchant bought goods at 25, 33⅓, and 10% off, and sold them for ¾ of the list price; what per cent did he gain?

39. A drover bought cows at $40 each: 20% of them having died, at what price per head must the remainder be sold to gain 20% on the entire cost?

40. B sold a horse at a loss of 25%: had the horse cost $120 less, he would have gained 25%; what did he receive for the horse?

41. The interest of a certain sum of money for 1 yr. 6 mo., at 6%, is $4.50; required the sum of money.

42. The money A paid for a horse, carriage, and sleigh would, in 8 years at 5%, give such an interest that, if put on interest for the same time and rate, it would amount to $252; how much did he pay for each, if the horse cost 3 times as much as the carriage, and the carriage twice as much as the sleigh?

43. A drover sold two horses at the same price, gaining 25% on the one, and losing 25% on the other; what was the cost of each horse, if he lost $16 by the transaction?

44. A bought goods to the amount of $416, on a credit of 6 months; what sum in cash will pay the debt, money being worth 8% ?

45. What is the amount of $600 for 4 yr. 6 mo. 12 da., at 5% ?.

46. What principal will in 4 yr. 6 mo., at 6%, amount to $381 ?

47. At what rate will $400 in 8 yr. 6 mo. give $272 interest?

48. A bought muslin at 6¼ cents a yard, and marked it at a gain of 20% : finding it damaged, he sold it at 75% of the marked price; what was the gain or loss per cent?

49. I sold goods at 5% through an agent who charged me 3% : my commission after paying the agent was $60; required my commission?

50. If $\frac{7}{18}$ of the price received for an article equal the loss, what is the loss per cent?

51. If oranges are bought at the rate of 8 for 5 cents, and sold at the rate of 5 for 8 cents, what is the gain per cent?

52. By selling 5 apples for 9 cents I lost 25% ; what per cent would I have gained by selling 4 apples for 10 cents?

53. If $\frac{3}{4}$ of the gain equal $\frac{3}{28}$ of the selling price, what is the gain per cent?

54. A sold two cows for $60 each, gaining 25% on one, and losing 25% on the other; what was the gain or loss per cent on the transaction?

55. An agent received $618 to purchase horses and cows; what sum can he invest in each, after deducting his commission of 3%, provided he invests twice as much for horses as for cows?

56. An agent sold 50 horses at $160 apiece, charging 5% commission: he invested the proceeds in sugar and cotton, charging 5⅚% commission for buying; how much did he invest in each, if ½ of the amount invested in sugar equals ⅓ of the amount invested in cotton?

57. What reduction from the price of an article are 30, 20, and 10% off?

58. What must a merchant ask for an article which cost $60, that he may give discounts of 20 and 10% off and still gain 20%?

59. Find the net amount of a bill of goods for $400, subject to discounts of 25, 20, and 16⅔% off.

60. What per cent above cost must goods be marked to allow a discount of 18¾% from the list price, and still make a profit of 17% ?

61. What per cent above cost must goods be marked to allow discounts of 25 and 20% off the list price, and still make 14% profit?

62. A's money is to B's as 9 to 8, and the interest on ⅓ of A's and ¼ of B's, for 4 years at 5%, is $1000; how much money has each?

63. M has ⅔ as much money as N, and the interest on ⅓ of M's and ¼ of N's money, for 2 yr. 6 mo. at 6%, is $225; how much money has each?

64. The interest on ⅔ of A's money and ¾ of B's, which equals ⅓ of A's, for 3 years at 5%, is $162; how much money has each?

65. The amount of a certain principal for 5 years at a certain per cent is $240, and the amount at twice that rate for 5 years is $320; required the principal and the first rate per cent.

66. The difference between the bank discount and the true discount of a certain sum of money, for 60 days at 6%, is 6 cents; required the sum of money.

67. The difference between the face of a note and its present worth, for 90 days at 6%, is $18; what is the face of the note?

68. The difference between the present worth and the proceeds of a certain sum of money, for 120 days at 6%, is $1; required the sum of money.

69. The difference between the bank discount and the true discount of a certain sum of money, for 90 days at 8%, is $2; required the sum of money.

70. What must I pay for 8% stocks to realize 5% on my investment?

71. What must I pay for 5% stocks to realize 8% on my investment?

72. A railway company declares a dividend of 4%; what is B's dividend, if he owns 120 shares of the stock?

73. When stock is selling at 25% premium, what amount can be bought for $5000?

74. B owns 300 shares of corporation stock: if the company declares a dividend of 7%, payable in stock, how many shares will he then own?

75. Three fifths of the cost of A's horse, plus ⅔ of the cost of his carriage, being on interest for 8 years at 5%, amount to $350; what was the cost of each, if ⅓ of the cost of the horse equaled ¼ of the cost of the carriage?

76. A received a stock dividend of 10%, and then owned 143 shares; how many shares did he own before the dividend was declared?

77. Two times A's money, plus 3 times B's, being on interest for 2 years at 5%, amount to $3960; how much has each, if 2 times A's equal 3 times B's?

78. Three times M's money, plus 2 times N's, being on interest for 4 years at 5%, amount to $54,600; how much has each, if 2 times M's equal 3 times N's?

79. If the interest on ⅓ of the cost of A's store, plus ⅓ of the cost of his house, for 8 years at 5%, be put on interest for the same time and rate, it will amount to $1680; what is the cost of each, if ¼ of the cost of the store equals ₅⁄₁₄ of the cost of the house?

80. James bought a watch for ⅝ of its value, and sold it for 25% more than its value; what was the gain per cent?

81. The amount of a sum of money for 10 years at 5% is $120 more than its amount for 8 years at 6%; what is the sum of money?

82. The interest of a sum of money for 4 years at 5% is $200 more than its interest for 3 years at 6%; what is the sum of money?

83. A merchant bought goods at 25% below cost, and sold them at 25% above cost; what amount of goods did he buy, if he gained $240? What was the gain per cent?

84. A merchant cheats his customers 12½% in both buying and selling, by means of false scales; what per cent does he gain fraudulently on goods bought and sold?

85. If the amount of $8000 for 10 years at 5% be divided in the proportion of 1 to 2, it will give respectively ⅓ of A's fortune and ½ of B's; required the fortune of each.

86. If the amount of $10,000 for 8 years at 5%, be divided in the proportion of ⅓ to ¼, it will give respectively ¼ of M's fortune and ½ of N's; required the fortune of each.

87. A merchant sold a piano for $600, at a gain of 20%, and received in payment a 60-day note, which he had discounted at a bank at 6%; what was his actual gain per cent?

88. A, B, and C, aged respectively 19, 17, and 15 years, inherited $3650, which was so divided that their several shares, at 10% simple interest, should amount to equal sums when they each arrived at the age of 21 years; what was the share of each?

89. What must be the face of a note which, when discounted at a bank for 90 days at 6%, will yield $985?

90. The amount of a certain principal for 8 years at a certain per cent is $560; and the amount at the same rate for 10 years is $600; required the principal and rate per cent.

91. A bought a horse on a credit of 9 months, but, by paying cash, received a discount of $12; required the cost of the horse, money being worth 8%.

92. A merchant asked 30% more for a piano than it cost: he afterward sold the piano for 10% less than his asking price, and gained $136 on the transaction; how much did he ask for the piano?

93. A farmer sold two cows at the same price, gaining 20% on the one, and losing 20% on the other; what was the cost of each cow, if he lost $6 by the transaction?

94. A and B together had $200: each gave 40% of his money to the other, after which A had $12 more than B; how much had each at first?

95. A sold two cows, and received 8% more for the second than for the first: on the first he gained 25%, and on the second he lost 10%; required the cost of each cow, if his gain on the transaction was $4.

96. A watch and chain were sold for $170,—the watch at a gain of 33⅓%, and the chain at a loss of 16⅔%; what was the cost of each, if the chain cost ⅔ as much as the watch?

97. B bought two bicycles for $40 and $60 respectively: he sold both at the same price, and gained as much per cent on the one as he lost on the other; what per cent did he gain or lose on the transaction?

98. A farmer asked 40% more for a wagon than it cost: he finally sold the wagon for 25% less than his asking price, and gained $7 on the transaction; how much did he ask?

99. A proposes to reduce the wages of his men 10%: his men "strike," but afterward submit on condition that their wages remain the same, and that their time, which is 10 hours a day, be increased; how much should it be increased to be equivalent to a reduction of 10% in wages?

100. A merchant marked coal so as to gain 40%, but, on account of using an incorrect weight, he gained only 25%; how many pounds did he give for a ton?

XIV. SUPPLEMENTARY PROBLEMS.

1. A POLE 130 feet high was made from two unequal pieces of timber, of which the shorter piece was ⅔ of the length of the longer piece, minus 10 feet; required the length of each piece.

2. A house and barn cost $10,000; what was the cost of each, if ⅘ of the cost of the house, plus $1000, equal the cost of the barn?

3. A hat, vest, and coat cost $27; what was the cost of each, if the vest cost $4 more than ½ of the cost of the hat, and the coat $10 more than ⅔ of the cost of the hat?

4. A, B, and C have $99: A has ⅔ as much as B, plus $12; and B has ⅔ as much as C, plus $18; how much has each?

5. A tree 102 feet high was broken into three unequal pieces: the first piece was ⅔ as long as the second, plus 10 feet; and the second was ⅘ as long as the third, minus 10 feet; what was the length of each piece?

6. Divide the number 91 into three parts, so that the first shall be 3 times the second, and the second 3 times the third.

7. A, B, and C together have $34,000; how much has each, if A has 3 times as much as B, and B 4 times as much as C?

8. A drover has 361 animals, consisting of horses, sheep, and cows; how many are there of each kind, if there are 3 times as many cows as horses, and 5 times as many sheep as cows?

9. In an orchard containing apple, peach, and pear trees, there are 2 times as many apple trees as pear trees, and 5 times as many peach trees as apple trees; how many are there of each kind, if there are 78 trees in the orchard?

10. In a certain library, ⅔ of the number of books on fiction equal the number on history, and ⅔ of the number on history equal the number on science; how many books in the library, if there are 2000 more books on fiction than on science?

11. If 8 horses eat a lot of grain in 12 weeks, in what time will the grain be consumed if 6 horses are added when the grain is ½ eaten?

12. If 10 men can do a piece of work in 12 days, in what time will it be completed if 2 men leave when the work is ¾ done? *| 3 𝒪.𝒶·|*

13. If 8 oxen can be kept on 4 acres for 3 months, how many sheep can be kept on 10 acres for 4 months if 6 sheep eat as much as 1 ox?

14. If 12 men dig a trench 8 rods long in 8 days, in what time can 18 rods be built if ¼ of the men leave when ½ of the work is done?

15. If 12 men can plow 4 acres in 3 days, how many boys can plow 16 acres in 4 days if 2 men do as much as 3 boys?

16. If 15 horses eat 12 bushels of oats in 10 days, how long will the oats last if ⅗ of the horses are taken away when ⅖ of the oats have been consumed?

17. If 27 men build 36 rods of wall in 60 days, in what time can 60 rods be built if ⅓ of the men leave when the work is ½ completed? *𝒷 b*

18. If 20 horses can be kept on 8 tons of hay for 6 months, how many cows can be kept on 12 tons for 4 months if 2 cows eat as much as 3 horses?

19. A farmer bought a horse, carriage, and harness for $210; what did he pay for each, if the cost of the horse is to the cost of the carriage as 3 to 1, and the cost of the carriage is to the cost of the harness as 5 to 1?

20. A pole 95 feet long was broken into three pieces: the lower part was to the middle part as 3 to 4, and the middle part was to the top part as 1 to 3; what was the length of each part?

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21. A and B can build a boat in 4 days, and A alone in 6 days; in what time can B alone build it?

22. A, B, and C can do a piece of work in 4 days: A can do it in 8 days, and B in 12 days; in what time can C alone do it? *2 4 𝒹𝒶𝓎*

23. D, E, and F can mow a field in 10 days, D and E in 15 days, and E and F in 20 days; how long would it take each to mow it? *3 0 b*

24. A and B can make a road in 8 days, B and C in 12 days, and A and C in 16 days; how long would it take each to do it?

25. A and B can do a piece of work in 4 days, A and C in 6 days, B and C in 8 days, and C and D in 12 days; in what time can each do it alone?

26. A and B can reap a field in 6 days, and B alone in 8 days; how long will it take A to finish it after both have labored 4 days?

27. If 4 men or 6 boys can do a piece of work in 10 days, in what time can 4 men and 6 boys do it? In what time can 6 men and 4 boys do it?

28. If 6 men or 8 boys can do a piece of work in 12 days, in what time can 3 men and 4 boys do it? In what time, 4 men and 3 boys?

29. C and D can do a piece of work in 8 days: after they both labor 5 days, C leaves, and D finishes it in 9 days more; in what time could each do the work alone?

30. D and E can make a road in 4 weeks, E and F in 6 weeks, and D and F in 8 weeks: after the three had labored 2¾ weeks, D left; how long did it require E and F to finish it?

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31. A steamboat descends a river at the rate of 18 miles an hour, and returns at the rate of 10 miles an hour; what is the rate of the current, and how far did the boat go, if it was gone but 7 hours?

32. Suppose that for every 4 cows a farmer has he should plow an acre of land, and allow 1 acre of pasture for every 3 cows; how many cows could he keep on 35 acres of land?

33. If a farmer allows 1 acre of corn for every 7 sheep, and 1 acre of grass for every 5 sheep, how many sheep can he keep on 24 acres?

34. A has 60 sheep: he allows 1 acre of corn for every 6 sheep, and 1 acre of pasture for every 5 sheep; how many acres does he own?

35. B has 70 cows: he allows 2 acres of corn for every 5 cows, and keeps 3 acres of grass for every 7 cows; how many acres has he?

36. A housekeeper has 24 boarders: she must buy each month 3 pounds of coffee for every 5 boarders, besides a waste of 1 pound for every 15 boarders; how many pounds of coffee are required each month?

37. A cistern which holds 35 barrels has two pipes: the first pours in a barrel in 3 minutes, and the second a barrel in 4 minutes; how long will it take to fill the cistern if both pipes flow at the same time?

38. A tank which holds 91 barrels has three pipes: the first pours in a barrel in 2 minutes, the second a barrel in 3 minutes, and the third discharges a barrel in 4 minutes; how long will it take to fill the tank if the three pipes are open at the same time?

39. A reservoir which holds 26 barrels has four pipes: the first pours in a barrel in 3 minutes, and the second a barrel in 4 minutes; but the third discharges a barrel in 5 minutes, and the fourth a barrel in 6 minutes; how long will it take to fill the reservoir if all the pipes are open at the same time?

40. How far is it from Philadelphia to New York, if I leave Philadelphia at 8.55 A. M., going at the rate of 40 miles an hour, and, after spending 2 hours in New York, return at the rate of 30 miles an hour, reaching Philadelphia again at 4.10 P. M.?

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41. A boy bought a number of lemons at the rate of 2 for 5 cents, and had 2 cents remaining; if he had bought the same number at the rate of 5 for 2 cents, he would have had 44 cents remaining; how many did he buy?

42. William bought an equal number of apples, pears, and peaches for 90 cents, giving 2 cents each for the apples, 3 cents each for the pears, and 4 cents each for the peaches; how many of each did he buy?

43. A drover bought pigs at $4 each, and twice as many sheep at $5½ each; how many did he buy of each kind, if they all cost $120?

44. M and N have each a certain sum of money: if M had $120 more he would have 4 times as much as N, but if he had $120 less he would have only ⅔ as much as N; how much money has each?

45. A and B have each a certain number of sheep: if A had 40 more, he would have ⅘ as many as B; but if he had 56 more, he would have 1½ as many as B; how many sheep has each?

46. A gentleman gave 99 cents to some poor children: to each girl he gave 8 cents, and to each boy 5 cents; how many were there of each, provided there were 5 times as many boys as girls?

47. B bought oranges at 3 cents each, and 3 times as many apples at 2 cents each; how many were there of each kind, if all cost 81 cents?

48. The owner of a factory pays his employés $68 a day, giving the boys $½, the women $1, and the men $1¼ a day; how many are there of each, if there are twice as many women as boys, and twice as many men as women?

49. A, B, and C built a house for $410: A received $1½, B $2¼, and C $3½ a day; how many days was each employed, if C was employed twice as many days as B, and B twice as many days as A?

XV. ADVANCED COURSE.

1. A RECEIVES $4 a day for his labor, and forfeits $1 each day he is idle; at the expiration of 50 days he receives $140; how many days was he idle?

SOLUTION.—Had he labored the whole time he would have received 50 times $4, or $200; he therefore lost by his idleness $200 minus $140, which are $60. Every day he was idle he lost $4, his daily wages, plus $1, his forfeit, which together are $5. If in 1 day he loses $5, to lose $60 it will require ⅕ of 60, = 12, or 12 days. Therefore, etc.

2. A man agreed to labor for $3 a day, on condition that every day he was idle he should forfeit $¾; how many days did he labor, if at the end of 60 days he received $120?

3. A boy agreed to carry 60 eggs to a certain store for ½ cent each, on condition that for each one he broke he should forfeit 2 cents; how many did he break, if he received 20 cents?

4. B receives $2½ a day for his labor, and pays 50 cents a day for his board; at the end of 60 days he has saved $100; how many days did he work?

5. A painter agreed to labor for $2¼ a day and his board, on condition that every day he was idle he should pay 60 cents for his board; how many days did he labor, if at the expiration of 60 days he received $78?

6. C receives $4½ a day for his labor, and pays $5 a week for his board; at the expiration of 12 weeks he has saved $201; how many days was he idle?

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7. Twenty-four persons engage a dinner; but before paying the bill 4 of the company withdraw, by which each person's bill is increased $⅖; how much do they pay for the dinner?

SOLUTION.—If the expense of 1 is increased $⅖, the expense of 20, the number that remained, is increased 20 times $⅖, or $8, which the 4 persons should have paid. If 4 persons pay $8, 24 persons should pay 6 times $8, or $48. Therefore, etc.

8. Twelve men hire a boat for sailing; but before starting 5 of the company decline going, by which the expenses of each are increased $5; how much do they pay for the boat?

9. Thirty persons charter a car for $600; but, by adding a certain number of persons to the company, each person's expenses are diminished $5; how many persons are added?

10. Sixty men chartered a car; but, by taking in 10 more persons, the expenses of each were diminished $1½; how much did they pay for the car?

11. Five men buy a factory; but, by taking in 2 more partners, the amount to be paid by each is reduced $5000; required the cost of the factory.   _____ .

12. What is the time of day if ½ of the time past noon equals the time to midnight?

SOLUTION.—Since ½ of the time past noon equals the time to midnight, ⅔, or the time past noon, plus ½ of the time past noon, or the time to midnight, equal ½ of the time past noon, or the time from noon to midnight, which is 12 hours; then the time past noon is ⅔ of 12 hours, or 8 hours; hence, it is 8 o'clock P. M.   Therefore, etc.

13. What is the time of day when ½ of the time past noon equals the time to midnight?

14. What is the time of day when ½ of the time past midnight equals the time to noon?

15. What is the time of day when twice the time past midnight equals the time to noon?

16. What is the time of day when ½ of the time past midnight equals the time past noon?

17. What is the time of day when ¼ of the time past midnight equals the time past noon?

18. What is the time of day when ½ of the time past noon equals the time past midnight?

19. What is the time of day when ½ of the time past midnight equals the time to midnight again?

20. What is the time of day when ½ of the time past midnight 2 hours ago equals the time to noon 2 hours hence?

21. What is the time of day when ¼ of the time past noon 3 hours ago equals the time to midnight 2 hours hence?

22. What is the time of day when ⅔ of the time past midnight equal ⅔ of the time to midnight again?

23. What is the time of day if ¼ of the time past 2 o'clock A. M. equals ⅓ of the time to 4 o'clock P. M.?

24. It is between 2 and 3 o'clock, and the hour- and minute-hands of my watch are together; what is the time of day?

SUGGESTION.—The minute-hand gains 11 spaces in going 12; to gain 1 space it must go 1⁄11 of 12 spaces, or 1 1⁄11 spaces; and to gain 2 spaces it must go 2 times 1 1⁄11 spaces, or 2 2⁄11 spaces, etc.

25. What is the time of day if it is between 3 and 4 o'clock, and the hour- and minute-hands of a watch are together?

26. What is the time of day if it is between 4 and 5 o'clock, and the hour- and minute-hands of a watch are opposite each other?

27. What time between 3 and 4 o'clock are the hands of a clock opposite each other?

28. What is the time of day if it is between 4 and 5 o'clock, and the hour- and minute-hands of a watch are 5 minutes of space apart? At what time are they 5 minutes of time apart? Is there more than one correct answer to either or both of these questions?

29. How long after 5 o'clock are the hour- and minute-hands of a watch 8 minutes of space apart? When, 8 minutes of time?

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30. A bought a number of hens for $12, and, having killed 5 of them, sold ⅓ of the remainder for cost, and received $3; how many hens did he buy?

SOLUTION.—If ⅓ of the remainder cost $3, the remainder cost $9; since they all cost $12, the 5 which he killed must have cost $12 − $9, or $3; if 5 hens cost $3, 1 hen cost $⅗; hence, he bought as many hens as ⅗ are contained times in 12, or 20. Therefore, etc.

31. B bought a number of sheep for $100, and, after killing 2 of them, sold ⅓ of the remainder for cost, and received $30; how many sheep did he buy?

32. C bought some wheat for $15, and, after using 6 bushels, sold ¼ of the remainder for cost, receiving $3; how much did he sell?

33. D bought a number of cows for a certain sum, and, 2 of them having died, he sold ¼ of the remainder for cost, and received $60, which were ⅓ of the cost of all; how many cows did he buy?

34. E, having bought a number of turkeys, killed 3 of them, sold ⅔ of the remainder for cost, and received $9, which was $18 less than all of them cost ; how many turkeys did he buy?

35. F bought a number of chickens for $30; but, after using 12 of them, he sold ¾ of the remainder, plus 6, for cost, and received $21 ; how many did he buy?

36. A lady purchased some silk for $72 : after using 12 yards of it, she sold ¾ of the remainder, minus 6 yards, for cost, and received $48 ; how many yards did she buy?

37. A farmer bought a number of sheep for $80 : after losing 4 of them, he sold 2 more than ¼ of the remainder for cost, and received $56 less than all of them cost; how many sheep did he buy?

38. A lost ¼ of his turkeys : now, if he sells ⅔ of the remainder for cost, he will receive $24; but if he kills 10, and sells ⅔ of the remainder for cost, he will receive $12; how many had he at first?

39. A farmer lost ⅕ of his sheep : now, if he finds 8 of them, and sells ⅓ of the remainder for cost, he will receive $64; but if he kills 8, and sells ½ of the remainder for cost, he will receive $64; how many had he at first?

40. M lost ⅔ of his hens : now, if he finds 12 of them, and sells ⅔ of the remainder for cost, he will receive $24; but if he kills 12, and sells ⅔ of the remainder for cost, he will receive $16; how many hens had he at first?

41. A farmer lost ⅖ of his ducks : now, if he finds 10 of them, and sells 4 more than ⅖ of the remainder for cost, he will receive $30; but if he loses 10 more, and sells 4 less than ⅖ of the remainder for cost, he will receive $10; how many ducks had he at first?

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42. A has $200, B and C have 6 times as much as A, and A and C have 4 times as much as B ; required the amount of money possessed by each.

SOLUTION.—Since B and C have 6 times as much as A, they have 6 times $200, or $1200, and they all have $1200 plus $200, or $1400. A and C have 4 times as much as B, which, added to B's, equal 5 times B's, which are what they all have, or $1400; hence, B has ⅕ of $1400, or $280, and C has $1200 minus $280, or $920. Therefore, etc.

43. D has $300, E and F have 5 times as much as D, and D and E have 8 times as much as F; how many dollars has each?

44. M has $5000, N and P have 7 times as much as M, and M and P have 4 times as much as N; required the fortune of each.

45. B has two watches and a chain: the first watch cost $80, the second watch and the chain cost 3 times as much as the first watch, and the first watch and the chain cost ⅔ as much as the second watch; required the cost of the second watch and of the chain.

46. A farmer has two horses and a saddle. The first horse is worth $360. If the first horse be saddled, it will be worth 5 times as much as the second; but if the second horse be saddled, it will be worth ½ as much as the first; required the cost of the second horse and the cost of the saddle.

47. A has two horses and a carriage: the first horse cost $140, the second horse and the carriage cost 3 times as much as the first horse; but the first horse and the carriage cost 6 times as much as the second horse; required the cost of the second horse and the cost of the carriage.

48. A has 12 sheep, B and C have 6 more than 4 times as many as A, and A and B have 6 less than 3 times as many as C; how many sheep has each?

49. R and S have 5 times as much money as T, lacking $110, and R and T have 8 times as much as S, plus $160; required the money owned by R and S, if T has $600.

50. A earned as much money as he had, and spent $12; he then earned as much as he had remaining, and spent $12 more, and then had $30; how much had he at first?

51. C has twice as much money as B, lacking $6, and B has twice as much money as A, lacking $8; how much each have A and B, if C has $18?

52. In May A earned as much money as he already had, and spent $20; in June he earned as much money as he had remaining, and spent $20; he did the same in July and August, and then had $100; how much had he at first?

53. D has ½ as much money as C, lacking $9; C has ½ as much as B, lacking $9; and B has ½ as much as A, lacking $9; how much each have A, B, and C, if D has $12?

54. A is 24 years old, and B is 6; in how many years will A be 3 times as old as B?

SOLUTION.—At the required time 3 times B's age will equal A's age; then 2 times B's age at the required time will equal the difference of their ages; but the difference of their ages is always 24 — 6, or 18, years; hence, B's age at the required time is ½ of 18 years, or 9 years. B is now 6 years old; hence, in 9 — 6, or 3, years A will be 3 times as old as B. Therefore, etc.

55. C is 35 years old, and D is 5; in how many years will C be only 6 times as old as D?

56. James is 30 years old, and John is 3; in how many years will James be only 4 times as old as John?

57. Arthur is 10 years old, and his mother is 4 times as old; in how many years will his mother be only twice as old?

58. Harold is 5 years old, and his teacher is 5 times as old; in how many years will his teacher be only 3 times as old?

59. B is 10 years old, and C is 40; how long since C was 6 times as old as B?

60. A is 30 years old, and B is 36; how long since A was ½ as old as B?

61. M is 5 times as old as N, and ¾ of the difference of their ages are 16 years; in how many years will M be 3 times as old as N?

62. ¼ of E's age is equal to ⅓ of F's, and the difference of their ages is 12 years; how long since ⅓ of E's age equaled ¼ of F's?

63. ¾ of M's age are equal to ⅝ of N's, and the difference of their ages is 4 years; how long since ⅝ of M's age equaled ¹⅜ of N's?

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64. A is 6 times as old as B, but in 2 years he will be only 4 times as old; how old is each at present?

SOLUTION.—By the first condition of the problem, 5 times B's age equal the difference of their ages; hence, B's age equals ⅕ of the *difference* of their ages. By the second condition of the problem, 3 times B's age equal the difference of their ages; hence, B's age equals ⅓ of the *difference* of their ages. Therefore, ⅓ of the difference minus ⅕ of the difference, or ²⁄₁₅ of the difference, are 2 years; and the difference of their ages is ¹⁵⁄₂ of 2 years, or 15 years. If 15 years are 5 times B's age, B is now 3 years old, and A is 6 times 3 years, or 18 years. Therefore, etc.

65. John is 5 times as old as James, but in 4 years he will be only 3 times as old; how old is each?

66. P is 8 times as old as Q, but in 1 year he will be only 7 times as old; required the age of each.

67. A's house is 4 times as old as his barn, but in 2 years the house will be only 3 times as old; required the age of each.

68. Harry is ¼ as old as his father, but in 40 years he will be ¾ as old; required the age of each.

69. Twelve years ago A was ¼ as old as B, but now he is ⁴⁄₉ as old; required the age of each.

70. Twenty-five years ago William was ¼ as old as his father, and 5 years ago he was ½ as old; how old is each at present?

71. Six years ago M was ¼ as old as N, but 6 years hence he will be ⁴⁄₉ as old; required the age of each at present.

72. Twenty-five years ago, when William married, he was 1¼ times as old as his wife; but 25 years hence he will be only 1¹⁄₁₅ times as old; what is the age of each at present?

73. When Andrew was married his wife's age was ⅝ of his, but 35 years later her age was ⁴⁄₅ of his; how old was each when they married?

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74. A farmer lost 12 sheep, and, after purchasing ⅔ as many as remained, found that he had ⁷⁄₈ as many as at first; how many had he at first?

SOLUTION.—After buying ⅔ as many as remained, he had ⁵⁄₃ of what remained, which equal ⁷⁄₈ of what he had at first; hence, the remainder equals ⅗ of ⅞, or ²¹⁄₄₀, of what he had at first. Then 12 sheep equal ⁴⁰⁄₄₀ − ²¹⁄₄₀, or ¹⁹⁄₄₀, of what he had at first; hence, he had at first ⁴⁰⁄₁₉ of 12 sheep, or 32 sheep. Therefore, etc.

75. A drover lost 16 sheep, and, after purchasing ⅔ as many as remained, found that he had ⁴⁄₉ as many as at first; how many had he at first?

76. Charles, having a certain sum of money, earned $30, and then spent ¼ of what he had. He had left 3 times as much as at first; how much had he at first?

77. B, having a certain amount of land, purchased 60 acres, and then sold ⅓ of all that he had. The land remaining was twice as much as he had at first; how many acres had he at first?

78. A boy spent $1.20, and then, earning ¾ as much as remained, found that he had ⅝ as much as at first; how much had he at first?

79. A and B entered into partnership: the first year they lost $400, and the next year gained 1½ times as much as remained, when it was found that they had doubled their money; what was the share of each, if A's stock was to B's as 2 to 3?

80. C, D, and E entered into partnership: the first year they gained $800, but the next year, after losing ⅘ of what they then had, it was found that they had only ⅗ of their original capital; how much stock had each at first, if C had ¾ as much as D, and D had ⅝ as much as E?

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81. A bought a certain number of cattle for $300: had he bought 3 more, at $8 less each, the lot would have cost him $366; how many cattle did he buy?

SOLUTION.—By the conditions of the problem, 3 more, at $8 less each, cost $366 — $300, or $66; and 1, at $8 less, cost ⅓ of $66, or $22. Hence, the first cost of each was $22 + $8, or $30; and for $300 he could buy 10 cattle. Therefore, etc.

82. A farmer sold a number of sheep for $240: had he sold 6 more, at $2 less each, he would have received $276; how many did he sell?

83. A drover bought a number of cows for $400: had he bought twice as many, plus 6, at the same price, they would have cost him $640 more; how many cows did he buy?

84. James sold a number of barrels of apples for $60: had he sold twice as many, minus 4, at the same price, he would have received $48 more; how many barrels did he sell?

85. William bought a number of hens for $6: had he bought 4 times as many, at 10 cents less each, they would have cost him $20; how many hens did he buy?

86. C bought a number of books for $24: if he buys 3 times as many more, at 50 cents more each, the cost of all will be $114; how many books did he buy at first?

87. M and N have equal sums of money: if N gives M ½ of his money, and M returns ¼ of what he then has to N, M will have $40 more than N; how much had each at first?

88. A and B had unequal sums of money: A gave B as much as he already had, then B gave A as much as A had remaining, when it was found that each had $20; how much had each at first?

89. B and C had unequal sums of money: B gave C as much as he already had, then C gave B as much as B had remaining, when it was found that each had $60; how much had each at first?

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90. A drover bought 22 sheep for $224: for the first he paid $12 each, and for the others $8 each; how many were there of each kind?

SOLUTION.—If all the sheep had been of the best grade, he would have paid $264, but he paid only $224; hence, he paid $40 less by buying sheep of the second grade; and since each sheep of the second grade cost $4 less than a sheep of the first grade, there were as many sheep of the second grade as 4 is contained times in 40, or 10 sheep. The number of sheep of the first grade was 22 − 10, or 12 sheep. Therefore, etc.

91. A lady bought 18 pounds of sugar for $1 : for the best she paid 6 cents a pound, and for the remainder 5 cents a pound; how many pounds did she buy of each kind?

92. A bought 25 ducks and turkeys for $18 : for the ducks he paid 60 cents each, and for the turkeys 80 cents each; how many were there of each?

93. A farmer bought 20 head of cattle for $460 : for the best he paid $25 each, and for the others $20 each; how many were there of each kind?

94. A merchant bought 50 knives for $16 : for the best kind he paid 35 cents each, and for the others 25 cents each; how many were there of each kind?

95. C bought 28 books for $62 : for the first kind he paid $2 each, and for the others $2½ each; how many were there of each kind?

96. A employed 30 laborers for $35 : the men received $2 each, and the boys $1 ; how many were there of each?

97. A merchant bought 50 tons of coal for $180 : for the best he paid $4 a ton, and for the remainder $3 a ton; how many tons were there of each kind?

98. A boy sold 70 oranges for $1.50: for the best he received 5 cents for 2, and for the others 5 cents for 3; how many did he sell of each kind?

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99. A has $30, B has as much as A plus $\frac{1}{4}$ as much as C; and C has 2 times as much as A and B together; how much has each?

SOLUTION.—$\frac{1}{4}$ of C's money, plus $30, equals B's, which, added to A's money, equals $\frac{1}{4}$ of C's money, plus $60; and this by the condition of the problem equals $\frac{1}{2}$ of C's money. If $\frac{1}{2}$ of C's money equals $\frac{1}{4}$ of C's money, plus $60, then $\frac{1}{2} - \frac{1}{4}$, or $\frac{1}{4}$, of C's money equals $60, and C's money is $360. B has $\frac{1}{4}$ of $360, plus $30, or $150. Therefore, etc.

100. M has $40, N has as much as M plus $\frac{1}{4}$ as much as P, and P has 3 times as much as M and N together; how much has each?

101. R has $20, S has as much as R plus $\frac{1}{3}$ as much as T, and T has as much as R and S together; how much has each?

102. Harold has $50, Oram has as much as Harold plus $\frac{1}{4}$ as much as John, and John has twice as much as Harold and Oram together; how much has each?

103. A pole is in the mud, air, and water: 5 feet are in the mud; the part in the water is as long as the part in the mud plus $\frac{1}{2}$ of the length in the air, and the part in the air is three times as long as the parts in the mud and water together; required the length of the pole.

104. In a company of men, women, and children there are 30 men: the number of women equals the number of men plus $\frac{1}{3}$ of the number of children, and the number of children is equal to the number of men and women together; how many persons in the company?

105. John earned $60, William earned as much as John plus $\frac{1}{4}$ as much as Henry, and Henry earned 3 times as much as John and William together; how much did each earn?

106. A has $100, B has as much as A plus $\frac{2}{3}$ as much as C, and C has $\frac{4}{5}$ as much as A and B together; how much has each?

107. In an alloy of 60 pounds there are 50 pounds of silver and 10 pounds of copper; how much copper must be added that there may be 2 pounds of copper to 5 pounds of silver?

SOLUTION.—If there are now 50 pounds of silver, which are 10 times 5 pounds, there must be 10 times 2 pounds, or 20 pounds, of copper, that there may be 2 pounds of copper to 5 pounds of silver; hence, there must be 20 — 10, or 10, pounds of copper added. Therefore, etc.

108. In an alloy of 50 pounds there are 40 pounds of gold to 10 pounds of silver; how much silver must be added that there may be 3 pounds of silver to 5 pounds of gold?

109. In a school of 120 pupils there are 3 boys to every 2 girls; how many girls must be received that there may be 2 boys to every 3 girls?

110. In a school of 80 pupils there are 3 boys to every 7 girls; how many girls must leave that there may be 3 boys to every 5 girls?

111. In a school of 100 pupils there are 4 boys to 1 girl; how many boys must leave the school that there may be 3 boys to every 2 girls?

112. In 120 pounds of alloy there are 5 pounds of silver to 3 pounds of copper; how much silver must be added that there may be 9 pounds of silver to 5 pounds of copper?

113. A pole 85 feet long is broken so that $\frac{2}{3}$ of the length of the longer equal $\frac{3}{4}$ of the length of the shorter piece; how much must be cut from the longer piece that $\frac{5}{8}$ of the shorter may equal $\frac{4}{7}$ of the longer?

114. In 150 pounds of alloy there are 8 pounds of gold to 7 pounds of silver; how many pounds of silver must be added that there may be only 7 pounds of gold to 8 pounds of silver?

115. If an alloy contain 48 ounces of silver and 3 ounces of copper, how much copper must be added that 36 ounces of the alloy may contain 30 ounces of silver?

116. If an alloy contain 60 ounces of gold and 3 ounces of silver, how much silver must be added that 22 ounces of the alloy may contain 20 ounces of gold?

117. A flag pole 130 feet high was broken into two unequal pieces, so that $\frac{1}{3}$ of the length of the first part equaled $\frac{1}{7}$ of the length of the second part; how much must be added to the second part that $\frac{1}{3}$ of the first part may equal $\frac{1}{8}$ of the second part?

XVI. MISCELLANEOUS REVIEW.

1. WHAT number exceeds the sum of its third, fourth, and fifth parts by 39?

2. A is five times as old as B, and in 10 years he will be only three times as old; what are their ages?

3. A is 72 years of age, and B is 52; how long since A was 6 times as old as B?

4. B had ⅓ as much money as C; but after C had given him $60, he had three times as much as C; how much had each at first?

5. M is 42 years older than N; and he is as much above 50 as N is below 36 years; required the age of each.

6. A, B, and C found a sum of money: it was agreed that A should receive $16 less than ½ of it, B $12 more than ¼ of it, and C the remainder, which was $29; how much did A and B receive?

\ *7.* A student spent $40 for books, ⅔ of the remainder of his money for clothing, and found that he still had ⅓ of all his money left; how much had he at first? ·

√ *8.* A farmer bought a number of chickens for $12: 4 having died, he sold ¾ of the remainder at cost, and received $7.20; how many chickens did he buy?

9. A bought 4 oranges, B 5, and C 7, and were then joined by D, who gave 12 cents; how should the money be divided if each ate an equal number of oranges?

10. Two sevenths of a flag-pole were broken off in a storm, and 30 feet more were cut off; the amount now taken off equaled the height of the pole after the storm; how high was the pole at first?

—*11.* A farmer plows 3 acres of land for every 11 cows that he owns, and allows 1 acre of pasturage for every 4 cows; how many cows can he keep on 69 acres?

12. Find the time between 7 and 8 o'clock when the minute-hand of a clock is 8 minute-spaces behind the hour-hand.

13. A boatman can row 8 miles an hour in still water; how far can he row down a stream which runs 3 miles an hour, that he may both go and return in 16 hours?

14. Sixteen men agree to dig a trench in 8 days: at the end of 4 days ¼ of the men leave; how many additional men must be employed at the end of the seventh day to complete the work in 8 days?

15. How far will a carriage have traveled when the fore wheel shall have turned 80 times more than the hind wheel, the circumferences being 6 and 8 feet respectively?

16. How much does a merchant, who gives ¼ of an ounce less than the legal standard in each pound, cheat a customer who buys goods amounting to $56?

17. A stone is weighed from both arms of a false balance, and its apparent weights are 16 pounds and 9 pounds respectively; find its true weight.

18. A and B enter into partnership, and gain $300: A owns ⅔ of the stock, lacking $50, and gains $175; required the entire stock.

19. A farmer bought a number of sheep for $120: had he then bought twice as many more, at $2 less each, all would have cost him $280; how many sheep did he buy?

20. A bought a number of pigeons for $16, and, having killed 4 of them, he sold ¼ of the remainder for cost, and received $3.60; how many did he buy?

21. M has $300, N and P have 4 times as much as M, and M and P have 3 times as much as N; required the funds of each.

22. A man drove to town at the rate of 4½ miles an hour, and returned at the rate of 6 miles an hour: he was detained in town 45 minutes, and was absent from home 5¹²⁄₁₃ hours; how far from town did he live?

23. A agreed to labor 120 days for $200 and a suit of clothes: at the end of 80 days he received $120 and the suit of clothes; what was the suit worth?

24. Ross is 6 years old, and his father is 42; in how many years will Ross be ⅓ as old as his father?

25. A man, dying, left a fortune of $38,000 to be divided among his son, daughter, and widow, on condition that if the daughter died, the widow should have ⅔ of the money; but if the son died, the widow should have ⅖ of it: they all survived; how should the fortune be divided?

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26. A steamboat sailed 100 miles and back in 20 hours; if it went 6 miles down the stream in the same time that it went 4 miles up, find its speed in going and returning, and the rate of the current.

27. One third of a number, multiplied by ½ of the number, is 1 less than the square of $\frac{1}{2}$ of the number; what is the number?

28. A person, being asked the time of day, replied that ½ of the time past noon, 2 hours ago, equaled ¼ of the time to noon again, 2 hours hence; required the time.

29. A has ¾ as much money as B: but if each had $20 more, A would have ¾ as much as B; how much money has each?

30. A grocer, by selling oranges at 30 cents a dozen, gained 42½ cents; but, had he sold them for 25 cents a dozen, he would have lost 12½ cents; what did the oranges cost him per dozen?

31. F lost 40 marbles: he afterward found ⅛ as many as remained, and then had ⅔ as many as at first; how many did he find?

32. If 42 pounds of sea-water contain 2 pounds of salt, how much salt must be added that 32 pounds of sea-water shall contain 2 pounds of salt?

33. A man left a fortune of $42,000 to his wife, son, and daughter, on condition that if the son died before coming of age the widow should have ⅓ of the fortune, but if the daughter died the widow should have ⅛ of it; required the share of each if they all live.

34. Why is the product of any two consecutive even numbers divisible by 8?

35. At what time between 10 and 11 o'clock are the hands of a watch at right angles with each other?

36. Two thirds of a mixture of wine and water are wine; but, when 10 gallons of water are added, the wine is only ½ of the whole; find the quantity of each at first.

37. A man can split a cord of wood in 2½ hours, and pile a cord in 40 minutes; how many cords can he split and pile in 38 hours of work?

38. A sold his horse for ¼ above cost: had the cost been ⅔ of what it really was, and the selling price remained the same, the gain would have been $72; find the first cost.

39. In a school of 84 pupils there are 7 girls to every 5 boys; how many girls must leave that there may be only 5 girls to every 7 boys?

40. In a mixture of silver and copper, consisting of 80 ounces, there are 5 ounces of copper; how much silver must be added that there may be ¼ of an ounce of copper to every 5 ounces of silver?

41. A pole 105 feet long was broken into two pieces, such that ⅓ of the shorter equaled ¼ of the longer piece; how much must be cut from the longer part, that ⅓ of it may equal ½ of the other part?

42. If 100 pounds of sea-water contain 4 pounds of salt, how much salt must be added that 4 pounds of the mixture shall contain ¼ of a pound of salt?

43. B and C enter into partnership and gain $360: B owns ⅔ of the stock, plus $40, and his share of the gain is $250; required the entire stock and the share of each.

44. M, N, and P agree to mow a field for $60: M mows ½ of the field, plus 4 acres, N mows ⅓ of the field, lacking 12 acres, and P mows the remainder; how many acres did each mow, if M receives $18 more than N?

45. A farmer bought a number of sheep, at $4 each, and had $32 remaining: if he had bought twice as many calves, at $6 each, he would have lacked $32 of having money enough to pay for them; required his sum of money.

46. A drover lost ⅔ of his sheep: now, if he finds 12, and sells 6 less than ⅔ of the remainder for cost, he will receive $120; but if he loses 12, and sells 6 more than ⅔ of the remainder for cost, he will receive $80; how many sheep had he at first?

47. Two thirds of the cube of a number are 10 more than the cube of ⅔ of the number; required the number.

48. A boy paid 56 cents for 22 apples: for some of them he gave 2 cents each, and for the others 3 cents each; how many were there of each kind?

49. James bought 23 fowls for $6.95, giving 25 cents apiece for ducks, and 40 cents each for turkeys; how many were there of each kind?

50. James is 30 years younger than his father, and ¾ of the father's age are 18 years more than the age of James; required the age of each.

51. R and S contribute $1200 for building a schoolhouse, which is to be located 2 miles from R and 3 miles from S; how much should each contribute, if they give in proportion to the reciprocals of their distances?

52. A man, being asked his age, said, "Two thirds of my age 4 years ago equal ½ of my age 8 years hence;" required his age.

53. Four men lunch together: A furnishes 5 cakes, B 7, and C 8, while D contributes 60 cents: how should the money be divided, if they share the cakes equally?

54. Two trains started at the same time from Philadelphia and New York, and proceeded toward each other at uniform rates: they met in 1 hour and 20 minutes, and the faster train had gone 30 miles farther than the slower train. It appears that it would take the faster train 40 minutes to travel the distance the other train had come, and 2 hours and 40 minutes for the slower train to travel the distance the faster train had come; required the distance from New York to Philadelphia, and the rate per hour of each train.

55. I must be at Harrisburg at a certain time: I find that if I walk at the rate of 6 miles an hour, I shall be 20 minutes too early, and if at the rate of 4 miles an hour, I shall be 25 minutes too late; how far have I to go?

56. A watch that loses 4 minutes in 15 hours is 6 minutes fast on Saturday noon; what is the correct time when it indicates 6 o'clock on Monday evening?

57. A man starts to walk from one town to another, at an average rate of 4 miles an hour. One half hour later his son starts on the same journey on a bicycle at the rate of 9 miles an hour. On reaching the second town the son rests ½ an hour, and, after riding 40 minutes, on his return trip, meets his father still on his way. Required the distance between the two towns.

58. A boatman can row up a stream a certain distance in 1 hour and 20 minutes, and back again in 1 hour; required the distance, if the rate of the current is 2 miles an hour.

59. A man sold a watch and chain for $115: on the watch he lost 33⅓%, and on the chain he gained 40%; did he gain or lose in the transaction, and how much, if ⅔ of the cost of the watch equaled $\frac{9}{10}$ of the cost of the chain?

60. The square of twice a number is 126 more than twice the square of ½ of the number; required the number.

61. Mr. Lindsay, after a tour in Europe, found that he had spent each day ¼ as many shillings as the entire number of days he was absent from home: his trip cost him £135; how many days was he away from home?

62. M sold two watches at the same price, gaining 25% on one, and losing 25% on the other: if he lost $20 by the transaction, what was the cost of each watch?

63. If I buy salt at $\frac{1}{2}$ of a cent a pound, and sell it in butter at 30 cents a pound, what is my gain per cent?

64. A cistern has 3 pipes: the first can fill it in 3 hours, the second can fill it in 4 hours, and, if all the pipes are open, the cistern will be filled in 45 minutes; in what time can the third pipe fill it?

65. A lady went to a store, and spent $\frac{1}{2}$ of her money and $1\frac{1}{2}$ more; at a second store she spent $\frac{1}{2}$ of the remainder and $1\frac{1}{2}$ more; and so on, at a third and fourth store, when she had but $10 remaining. How much had she at first?

66. The fence around a square field cost $80, at 50 cents a rod; how many acres in the field?   '

67. What is the greatest number that will divide 135, 111, and 99, and leave the same remainder in each case?

68. What is the greatest number that will divide 143, 161, and 188, and leave the same remainder in each case?

69. The contents of a rectangular bin are 3750 cubic feet; what are its dimensions if its depth, breadth, and length are in the ratio of 2, 3, and 5 respectively?

70. Two thirds of the square of a number, less 4, equal the square of $\frac{2}{3}$ of the same number, plus 4; what is the number?

71. The square of the sum of two numbers is 196, and the cube of their difference is 8; required the numbers.

72. The product of four numbers is 150: two of the numbers are 2 and 3; what are the other two numbers, if they are equal?

73. The product of four numbers is 864: one of the numbers is 4; what are the others, if they are equal?

74. Two thirds of a certain number, multiplied by $\frac{3}{4}$ of the same number, are 28 less than the square of $\frac{5}{8}$ of the number; what is the number?

75. A person being asked the time of day, said, "One third of the time past midnight, 5 hours ago, equaled $\frac{3}{4}$ of the time to midnight, one hour hence;" required the time of day.

76. A bought 7 oranges, B 9, and C 12: they were joined by D, who gave the others 35 cents; how should the money have been divided, if each ate an equal number of oranges?

77. A and B are a mile apart, and travel toward each other: A takes 5 steps while B takes 4, but 2 of B's steps equal 3 of A's; how many rods will each have traveled when they meet?

78. A bought plums at 3 for a cent, and 3 times as many at 4 for a cent: he sold all of them at 3½ cents a dozen, and gained 10 cents; how many did he buy?

79. A and B gained $800 in trade: A put in $200 for 6 months, and B $250 for 8 months; what was each man's share of the gain?

80. The entire surface of a cube is 384 square feet; required the length of the edge.

81. A spent $50 for a bicycle, ¼ of the remainder of his money for books, ¼ of what now remained for clothing, and found that he then had just ¼ of his original sum: how much money had he at first?

82. Find the edge of a cube in feet whose surface and contents are *numerically* equal.

83. Find the length of a square field that contains as many acres as there are rods in its side.

84. How many three-inch cubes can be placed in a cubical box one yard deep?

85. What are the dimensions of a rectangular field that contains 486 square rods, if the width is to the length as 2 to 3?

86. M and N, aged respectively 19 and 17 years, inherited $13,000, which were so divided that their shares, at 10% simple interest, should amount to equal sums when they are 21 years of age; what was the share of each?

87. A, B, and C, aged respectively 17, 13, and 11 years, inherited $9300, which were so divided that their several shares, at 5% simple interest, should amount to equal sums when they are 21 years of age; what was the share of each?

88. E is 35 steps ahead of F, and takes 4 steps while F takes 3, but 2 of F's steps are equal to 5 of E's; how many steps will E take before he is overtaken by F?

89. R takes 75 steps before he is overtaken by S; how far ahead of S was R when they started, if R takes 3 steps while S takes 4, and if 5 of S's steps are equal to 4 of R's?

90. N can beat M ¼ of a mile in a 5-mile race: P can beat N by 6 minutes, and M by one mile, in the same course; find the rate of each per hour.

XVII. THE METRIC SYSTEM OF WEIGHTS AND MEASURES.

THE **Metric System** of weights and measures originated in France in 1795, and was adopted by the leading countries of Europe and South America, in many of which it was made compulsory. In 1864, Great Britain by an act of Parliament permitted its use throughout the kingdom, and in 1866 Congress legalized its use in the United States. Although the system is very simple, it has not been generally adopted in this country, its use being confined principally to the sciences and to the Coast Survey, and to some extent, also, in the Mint and the General Post Office.

The principal metric units are—

1. The **Meter,** for lengths.
2. The **Are,** or square decameter, for surfaces.
3. The **Stere,** or cubic meter, for volumes.
4. The **Liter,** or cubic decimeter, for capacities.
5. The **Gram,** or cubic centimeter, for weights.

The *Lower Denominations* are expressed by writing before the name of the unit the prefixes—

Milli—, to denote *one-thousandth* of the unit.
Centi—, to denote *one-hundredth* of the unit.
Deci—, to denote *one-tenth* of the unit.

The *Higher Denominations* are expressed by writing before the name of the unit the prefixes—

Deca—, to denote *ten* times the unit.
Hecto—, to denote *one hundred* times the unit.
Kilo—, to denote *one thousand* times the unit.
Myria—, to denote *ten thousand* times the unit.

Measures of Length.

The *Meter*, the principal unit of length, is very nearly one ten-millionth of the distance from the equator to the pole.

TABLE.

10 millimeters (mm.)	=	1 centimeter,	cm.
10 centimeters	=	1 decimeter,	dm.
10 decimeters	=	1 meter,	m.
10 meters	=	1 decameter,	Dm.
10 decameters	=	1 hectometer,	Hm.
10 hectometers	=	1 kilometer,	Km.
10 kilometers	=	1 myriameter,	Mm.

APPROXIMATE VALUES.

The *Meter*, about 3 ft. 3⅜ in., is used in measuring short distances.
The *Kilometer*, about ⅝ of a mile, is used in measuring long distances.
The *Decimeter* is about 4 inches long.

NOTE.—The pupil should fix these approximate values in mind by means of concrete illustrations.

How many—

1. Meters in a kilometer? Hectometer? Myriameter?
2. Millimeters in a meter? Decimeter? Centimeter?
3. Centimeters in a meter? Decameter? Hectometer?
4. Decimeters in a meter? Decameter? Kilometer?
5. Decameters in a hectometer? Kilometer? Myriameter?

Approximately—

6. How many inches in a meter?
7. How many decimeters in a foot?
8. How many kilometers in 10 miles?
9. How many miles in a myriameter?
10. How many miles in 24 kilometers?

Measures of Surface.

The *Square Meter* is the unit of area.

TABLE.

100 sq. millimeters (sq. mm.)	=	1 sq. centimeter,	sq. cm.
100 sq. centimeters	=	1 sq. decimeter,	sq. dm.
100 sq. decimeters	=	1 sq. meter,	sq. m.
100 sq. meters	=	1 sq. decameter,	sq. Dm.
100 sq. decameters	=	1 sq. hectometer,	sq. Hm.
100 sq. hectometers	=	1 sq. kilometer,	sq. Km.
100 sq. kilometers	=	1 sq. myriameter,	sq. Mm.

The following are also used:

.100 centiares   **=**   1 are.

100 ares   **=**   1 hectare.

APPROXIMATE VALUES.

The *Square Meter*, about 10¾ sq. ft., is used in measuring ordinary surfaces.

The *Square Decameter* or *Are*, about 4 sq. rd., and the *Hectare*, about 2½ acres, are used in measuring land.

How many—

   1. Sq. meters in a sq. decameter? In a sq. hectometer?

   2. Sq. centimeters in a sq. decimeter? In a sq. meter?

   3. Sq. decameters in a sq. hectometer? In a sq. kilometer?

   4. Sq. decimeters in a sq. meter? In a sq. decameter?

   5. Sq. millimeters in a sq. centimeter? In a sq. decimeter?

Approximately—

   6. How many sq. ft. in 8 sq. meters?

   7. How many acres in 10 hectares?

   8. How many sq. rd. in 25 ares?

   9. How many hectares in 20 acres?

   10. How many ares in 6 rods square?

   11. How many ares in an acre?

Measures of Volume.

The *Cubic Meter*, called the *Stere*, is the unit of volume.

TABLE.

1000 cu. millimeters (*cu. mm.*)   **=**   1 cu. centimeter, *cu. cm.*

1000 cu. centimeters   **=**   1 cu. decimeter, *cu. dm.*

1000 cu. decimeters   **=**   1 cu. meter,   *cu. m.*

APPROXIMATE VALUES.

The *Cubic Meter*, about 1⅓ cu. yd., is used in measuring ordinary solids.

A *Cord* is about 3¾ steres.

   1. How many cu. millimeters in a cu. decimeter?

   2. How many cu. centimeters in a cu. meter?

3. How many cu. millimeters in a cu. centimeter?

4. How many cu. centimeters in a cu. decimeter?

5. How many cu. millimeters in a cu. meter?

Approximately—

6. How many cu. yards in 12 cu. meters?

7. How many cords in 22 steres?

8. How many cu. meters in a cellar 15 ft. long, 12 ft. wide, and 3 ft. deep?

9. How many cords in a pile of wood 22 meters long, 1 meter high, and 1 meter wide?

10. How many steres in 60 cu. yd.?

Measures of Capacity.

A *Cubic Decimeter*, called a *Liter*, is the unit of capacity.

TABLE.

10 milliliters (ᵐˡ·)	=	1 centiliter,	cl.
10 centiliters	=	1 deciliter,	dl.
10 deciliters	=	1 liter,	l.
10 liters	=	1 decaliter,	Dl.
10 decaliters	=	1 hectoliter,	Hl.
10 hectoliters	=	1 kiloliter,	Kl.
10 kiloliters	=	1 myrialiter,	Ml.

APPROXIMATE VALUES.

The *Liter*, about $1\frac{1}{18}$ liquid quarts, or $\frac{9}{10}$ of a dry quart, is used in measuring liquids and small fruit.

The *Hectoliter*, about $2\frac{5}{8}$ bu., is used in measuring grain, vegetables, etc.

How many—

1. Centiliters in a liter? In a decaliter?

2. Deciliters in a decaliter? In a hectoliter?

3. Liters in a hectoliter? In a kiloliter?

4. Centiliters in a hectoliter? In a decaliter?

5. Milliliters in a liter? In a decaliter?

Approximately—

6. What part of a hectoliter is a bushel?

7. How many liters in a liquid gallon?

8. How many liters in 38 liquid quarts?
9. How many hectoliters in 34 bushels?
10. How many liters in 36 dry quarts?

Measures of Weight.

The weight of a *Cubic Centimeter* of distilled water at its greatest density (39.2° Fahrenheit), called a *Gram*, is the unit of weight.

TABLE.

10 milligrams (ᵐᵍ·)	=	1 centigram,	ᶜᵍ·
10 centigrams	=	1 decigram,	ᵈᵍ·
10 decigrams	=	1 gram,	ᵍ·
10 grams	=	1 decagram,	ᴰᵍ·
10 decagrams	=	1 hectogram,	ᴴᵍ·
10 hectograms	=	1 kilogram,	ᴷᵍ·
10 kilograms	=	1 myriagram,	ᴹᵍ·

APPROXIMATE VALUES.

The *Gram*, about $\frac{1}{28}$ of an ounce Troy, is used in weighing precious metals, jewels, and letters, and in mixing medicines.

The *Kilogram*, about 2⅕ pounds avoirdupois, is used in weighing ordinary articles.

The *Metric Ton*, about 2200 lb., is used in weighing heavy articles.

How many—

1. Grams in a hectogram? In a kilogram?
2. Decagrams in a kilogram? In a myriagram?
3. Decigrams in a decagram? In a hectogram?
4. Centigrams in a hectogram? In a decagram?
5. Hectograms in a kilogram? In a myriagram?

Approximately—

6. How many ounces in 112 grams?
7. How many pounds in 10 kilograms?
8. How many tons in 10 metric tons?
9. How many kilograms in 44 pounds?
10. How many grams in 10 ounces?

Measures of Value.

The *Franc* is the unit of French money. Its value is $0.193.

TABLE.

,   10 centimes (c.)   −   1 franc, f.

The coins of France are—

Gold—the *five-franc, ten-franc, twenty-franc, forty-franc,* and *hundred-franc* pieces.

Silver—the *twenty-five-centime, fifty-centime, franc, two-franc,* and *five-franc* pieces.

Bronze—the *one-centime, two-centime, five-centime,* and *ten-centime* pieces.

Miscellaneous Problems.

1. How many decimeters in a yard?

2. How much is gained by buying 10 meters of cloth at $2 a meter, and selling it at 25 cents a decimeter?

3. A bought 5 hectoliters of chestnuts at $7 a hectoliter, and sold them at 10 cents a liter; required the gain.

4. How much is gained by buying a barrel containing 15 myriagrams of sugar for $12, and selling it at 12 cents a kilogram?

5. An opera-glass cost 100 francs; required its cost in United States money.

6. A horse traveled 8 kilometers in an hour; how far at this rate can he travel in 10 hours?

7. If a horse eats 2 decaliters of oats in a week, how long will 2 hectoliters last him?

8. How much is gained by buying 30 steres of wood at $1 a stere, and selling it at $5 a cord?

9. B bought a farm containing 20 hectares at $200 a hectare, and sold it at the rate of $8 an are; required his gain.

10. C bought a barrel of vinegar containing 2 hectoliters for $8, and sold it at 6 cents a liter; required his gain.

Those evening bells. ring out their
sweet chime How many a tale
their music tells of youth and
home and that sweet chime
at last I heard those evening
bells.

Mildred
Newitt